A Social Space -Time Dynamic Model as a Basis of International Network Theory

Paul R. Williamson
Global Vision, Inc.
paulrw@globechange.org
www.globechange.org

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Abstract.

A characteristic pattern, which I call the nodal pattern, describes certain kinds of international interactions for selected recent time periods; it is plausible but not yet investigated that this pattern is generally descriptive for all modern time periods. The behaviors in question include merchandise exports, importation of weapons, and hosting of foreign troops on national territory. The pattern, itself, consists, first, of a marked preference for all other nations to concentrate the above behaviors on a handful of named nations, the same nations in each instance. For the period investigated, these latter nations were United States, Russia, United Kingdom, France, and Japan. Second, the pattern exhibits (in the data shown here to modest degree) a bimodal fracturing of the named nations into two groups, such that other nations tend to concentrate on one or more nations of one group or the other. This fracture line was between United States, United Kingdom, France, and Japan, on one hand, and Russia on the other.

The above ideas are interesting and suggestive of further (and more thorough and systematic) empirical work; however, the focus in the present paper is on the ability of a particular dynamic mathematical model to replicate the nodal pattern, at any one moment, and its changes over past historical moments, i.e. its dynamics. The mathematical model in question is a further development and modification of the social field theory proposed several decades ago by Quincy Wright (1955) and further developed by Rummel (1965). This same model can suggest connections from the nodal pattern to several other phenomena and directions of inquiry. The latter include: arms race modeling, "power transition" and global war, spheres of influence and bimodal polarization, and rational choice modeling. A complimentarity between utility maximizing and dynamics becomes evident.

Put most simply in terms of this model, structural phenomena such as global spheres of influence and bipolarity are seen as byproducts of an arms-race-like interaction between the internal development processes of nations and other social units.

Continued next page.

Table of Contents.

Introduction.

We begin by discussing the arms race model introduced by Richardson (1960). We will consider a way of deriving this model from certain assumptions by combining, with slight modification, ideas described by Boulding (1963) and McGuire (1965), and using assumptions about the “utilities” of choice theory. We will then see that utilities and “dynamics”—principles by which a system changes from one moment to the next—have a complimentary mutual relationship.

Next, we will develop a connection between Richardson’s model and the social field theory of Wright (1955). We will do this by connecting both of them to a dynamic social model adapted from the mathematical framework of electrodynamics. This framework is that of special relativity, which will be seen to correspond, in the dynamic social model, to the possibility of a form of “political polarization” that distinguishes between “central” (“major”) and “peripheral” (“minor”) powers in world politics. The electrodynamic analogue will be seen to point, also, to the possibility of building a quantitative, empirical, computational model of a “political” aspect of world history.

In the following, to assist in locating first definitions or other identification of terms and quantities, I underline them.

1. Composition Assumption.

Richardson started with the idea that arms racing may be measured in various alternative empirical forms: as military expenditure, as some quantitative expression of the resulting capabilities of armaments, as the number of person-hours devoted to producing and using armaments, and so on. As to which of them to use, we omit from this discussion.

We also need to use some care in selecting terminology. Herein, we will talk non-specifically about the quantity of "provocative activity" by one party (or social unit—the two terms will be used interchangeably), in response to some combination of a) its own inclinations, b) one or more other parties, and c) conditions in the environment. For brevity, in the following we will refer to this quantity of activity as the "provocation" by the given party. This activity is provocative in the sense that it leads to responses in the form of like activity by other parties; it need not necessarily exhibit any overt communication of hostility, thus the quotation marks. I will refer to each such response, by a given party to the provocation of a specific other party, as a partial response which, when the meaning is clear from context, I will shorten to "response". I use this language to reflect the idea that the total (quantity of) response of the given party is a composite of all its partial responses to the provocations of other parties. To complete the set of ideas, this total response constitutes, in turn, the provocation by the given party to which all others subsequently give their partial responses.

We start by introducing the assumption that the total response of a specific party equals the product of its partial responses to other parties. If r_i represents the total response of party i, r_{ij} the (quantity of) response to the j^{th} other party, and N the total number of parties, then this composition assumption is given by

$$r_i = \Pi_{j \neq i,\, j=1}^{N} r_{ij} \tag{1.1}$$

Notice this says that a greater partial response to a given other party always contributes to a greater total response. In the above, we do not know what a "negative" amount of partial response means; it remains undefined. Moreover, let us assume that always there is some non-zero degree of response between any two parties.

[One can also put this assumption in the following different way: Suppose the partial response of a given party to a certain other party P is zero. In that case, the convention would be to leave P out of the definition of the "system" of parties that is being modeled. For equation (1.1) to be consistently applied to all responding parties, however, it would need to be also true that no other party in the system responds partially to P, either (in which case P simply would not be regarded as part of the system). While one could take the point of view that the latter is a substantive assumption that might be empirically false, the point of view taken here is that partial responses are not directly observed—only the total response to all others is observed. So, in the indicated circumstance, one is free to assign a value. Here we choose to assign the value $r_{iP} = 1$ to signify that there is no partial response to P. Since the partial responses combine multiplicatively in equation (1.1), assigning the value 1 amounts to saying, not unreasonably, that such a "no partial response" does not alter the total response. Later, starting with equations 3.1, we will consider the logarithms of partial responses; the preceding equation will then become $\ln r_{iP} = 0$, which value accords more conventionally with no response.]

To reflect these ideas, we further constrain the relevant values by

$$r_{ij} > 0 \;,\; i, j = 1,2,\ldots,N \;,\; i \neq j \tag{1.2}$$

From equations (1.1) and (1.2), we conclude that

$$r_i > 0 \tag{1.3}$$

2. Utilities of Partial Response.

McGuire (1965) discusses how a particular economic framework for describing price levels between two duopolists can also describe armament levels in a competition between two parties. Here, I use this adaptation in the same way McGuire did, to describe a relationship between the total provocation of one party and the partial response (my terminology, not his—see above) of another. (Rather than collaborating to raise price, as in a duopoly, the parties might here collaborate to reduce the provocations that are expressed as their military efforts, that is they might engage in arms control; however, I will not develop that aspect here.) For simplicity we first talk in terms of parties labeled “party 1” and “party 2”, though it will be clear that the argument applies to any two parties. Later, we generalize the labeling to any two parties such as “i” and “j”.

In Figure 2.1 (following the bibliography), the vertical axis denotes the total provocation r_2 of party 2; the horizontal axis denotes the partial response $r_{1,2}$ of party 1 to r_2. (The numbers are dimensionless and for illustration, only.) So, Figure 2.1 is saying that a given total provocation by party 2 may be associated with a certain partial response by party 1.

That the party 2 level, to which party 1 is responding, is the total level, reflects the assumption that, to party 1, all provocative aspects of party 2 appear equally and indiscriminately so. (The possibility of “direction” effects, for instance that a given body of army units would be more provocative to a neighbor if deployed on their common border, might be represented in the values of the “reaction coefficients” that are discussed below. Herein I omit this admittedly important issue.)

Combinations of the provocation by party 2 with the various possible corresponding partial responses of party 1, are depicted by points plotted in the $(r_{1,2}, r_2)$-plane. (Following standard economics format, the independent variable—namely, the provocation of party 2—is on the vertical axis in this particular figure and in Figure 4.1, below. Points having *equal* utility to party 1 are connected by the curved lines, such as the two shown (marked “u-curve...”), assumed to be everywhere concave downward. This assumption has the following interpretation: At small levels, small increases in the value of r_2 can be compensated by small increases in $r_{1,2}$; that is, the increased “threat” to party 1 can be exactly offset by increased “defenses”. Beyond the point marked M (for “maximum”), however, the marginal unit cost—always increasing—of further defensive increases outweighs the marginal security benefit.

If one considers the whole family of such utility curves, the direction of their increasing utility is downward, reflecting the idea that, for a given partial response $r_{1,2}$ (corresponding to a given armaments level, etc.), party 1 would prefer a lesser, over greater, level of provocation r_2 by party 2. This implies that, for a given value r_2, the maximum utility available to and, thus, the optimum response of, party 1 is at the point M. (From what has just been said, this implication follows from the fact that the horizontal line passing through r_2 on the vertical axis will encounter only the constant utility contour on which is located M, plus other constant utility contours having lesser utility values.)

Due to the everywhere downward concavity assumed of the constant utility curves, this maximum is global as well as local. Thus, for given r_2, M is unique. Thus the value of $r_{1,2}$ at point M, let us call this value $r_{M,1,2}$, is a function $f(r_2)$ of the provocation level of party 2. Let us assume further that this function is differentiable, so that

$$f'(r_2) \equiv \frac{df(r_2)}{dr_2} = \frac{dr_{M,1,2}}{dr_2} \qquad (2.1)$$

exists for all values of r_2 within the domain of the original function f and, also, that

$$f'(r_2) > 0 \qquad (2.2).$$

This assumption has the meaning that, as the provocation level of party 2 increases, the corresponding partial response level of party 1 also increases. (Here and below the notation f', or with f replaced by any other letter that names a function, followed by the prime mark, will refer to the derivative of the named function with respect to its argument.)

3. Reaction Curve.

In the above, point M is specific to the provocation level r_2 of party 2. Now let us consider the collection of points such as M_1, M_2, M_3**...**, each one representing an optimum response of party 1 to corresponding varying provocation levels of party 2. From the previous discussion we know that the coordinates of these points will be described by the function f which, by equation (2.1), must be continuous (because f is differentiable); thus we know that M_1, M_2, M_3 **...** must be connected by a smooth line.

Next let us define new variables $z_{1,2}$ and z_2 given by

$$z_{1,2} \equiv \ln(r_{1,2})$$
$$z_2 \equiv \ln(r_2) \qquad (3.1).$$

Imagine that the smooth line connecting M_1, M_2, M_3 **...** is re-plotted on a log-log graph of $r_{1,2}$ versus r_2; that is, this new graph will depict $z_{1,2}$ versus z_2 (shown in Figure 4.1, below). From equations (1.2) and (1.3), $z_{1,2}$ and z_2 are well defined. In this new graph, the quantity $r_{M,1,2}$

appearing in equation (2.1) will be plotted as $\ln(r_{M,1,2}) \equiv z_{M,1,2}$. This log-log transformed version of the original smooth line will have a derivative given by

$$\begin{aligned} z'_{M,1,2} &\equiv \frac{dz_{M,1,2}}{dz_2} = \frac{d[\ln(r_{M,1,2})]}{d[\ln(r_2)]} \\ &= \frac{(1/r_{M,1,2})dr_{M,1,2}}{(1/r_2)dr_2} \\ &= (r_2/r_{M,1,2})f'(r_2) \end{aligned}$$

(3.2),

where the last expression is found using equation (2.1). Thus the originally smooth line is also smooth in its re-plotted form. Let us denote this re-plotted line by L , which we will call the *partial reaction curve* of party 1 to party 2. (This curve is "partial" in two senses: first, as mentioned above, the total response of party 1 includes the effects of other parties beside party 2; second, with respect to this total response, the "final" or equilibrium responses result from interaction with the corresponding total response for party 2.)

In general, L is not necessarily a straight line; thus the derivative given in (3.2) may vary, depending on the point on L to which the derivative corresponds; that is, depending on z_2. To anticipate what follows below, we can imagine that, in a realistic application, z_2 itself varies with time t ; thus the derivative is also functionally dependent on time. This can be expressed by writing $z'_{M,1,2}$ of equation (3.2) as $z'_{M,1,2}[z_2(t)] \equiv z'_{M,1,2}(t)$ which can be further abbreviated as the expression on the right. [Note that the above does *not* equal the derivative with respect to time $\dot{z}_{m,1,2} \equiv \frac{dz_{m,1,2}}{dt} = \{z'_{M,1,2}[z_2(t)]\}\frac{dz_2}{dt}$.] Depending on need, we will express this derivative both with and without t . From equations (1.2), (1.3), (2.2) and (3.2), we conclude that

$$z'_{M,1,2}(t) > 0$$

(3.3).

<u>4. Rate of Convergence to Partial Equilibrium.</u>

To summarize the discussion so far, the partial reaction curve L reflects the greatest utility available to party 1 as it faces a given level of provocation by party 2. Let us assume that party 1 will seek to move toward its optimum response, the point M ; that is, it will move horizontally toward the value $z_{M,1,2}$ lying on its partial reaction curve L. That statement summarizes the *utilities* of the situation. The *dynamics* of the situation is reflected in the following question: *How quickly* will party 1 move toward its partial equilibrium? Or, more precisely, *what is the quantitative form* of that movement over time?

From the above comments it is evident that utilities are not the whole story—utilities and dynamics are *distinct* issues. Let us address the second question by making the following assumption (Boulding, 1963, p.28, note 5): the time rate of change $\dot{z}_{1,2} \equiv dz_{1,2} / dt$ in the partial reaction $z_{1,2}$ of party 1 is given by

$$\dot{z}_{1,2} = k_1(t)\Delta z_{1,2} \ , \ k_1(t) < 0 \tag{4.1}$$

where k_1 is a function of time assumed specific to party 1 and $\Delta z_{1,2}$ is defined by

$$\Delta z_{1,2} \equiv z_{1,2} - z_{M,1,2} \tag{4.2}$$

That is, the rate of convergence of the partial reaction $z_{1,2}$ of party 1 is assumed to be proportional to the distance from its own partial reaction curve L , as shown in Figure 4.1; thus utilities and dynamics are complimentary in the sense that *both ideas are needed to furnish the complete meaning* of equation (4.1). We can also see that the negativity of k_1 makes the process convergent rather than divergent.

In effect, (4.1) reflects the idea that party 1 cannot respond instantaneously to changes in the provocation level of party 2 but can only adjust its partial reaction at some finite rate given by k_1 . The party 1 subscript reflects the further assumption that this rate of adjustment is the same for the responses of party 1 to all others. As written, this rate of adjustment may vary with time. In addition, other quantities are introduced below as time varying but as generalizations of quantities originally (in treatments by other writers) regarded as constants. In Part 6, we will encounter an approach originally developed in a way that is dependent on making these assumptions of constancy at all times for a given party. In Part 7 we will show what happens when these assumptions are dropped, and k_1 , and the other quantities are allowed to vary over time which, it will turn out, is required for the intended application.

One further point, implicit in the above, should be made explicit. Just as for $z'_{M,1,2}$ in equation (3.3), since the partial reaction quantities are functions of time, a more complete notation would be to write them as $z_{1,2}(t)$. Where no confusion would result, the t -argument will be omitted (as above); at other places, where needed, it will be restored. The same remarks apply to any other time-varying quantity, including any quantity derived or constructed from the $z_{1,2}(t)$.

5. Dynamics of Response of Party 1.

Now let us look at the above from a slightly different viewpoint. We will use this different viewpoint to obtain the dynamics of the partial response of party 1 to the provocation of party 2; then we will develop the response of any party to all others.

In Part 3 the value of $z'_{M,1,2}$ was found to be given by equations (3.2) and (3.3). The existence of $z'_{M,1,2}$ shows that, for any given value of z_2 , the straight line tangent to L at the corresponding point M exists. Let this tangent line be denoted by $R_M(t)$ where functional dependence on time is indicated in accordance with the earlier discussion leading to equation (3.3). What is the slope of this tangent line?, is a potentially confusing, slightly tricky, question. If we were to adhere to the convention that the "run" of a slope is provided by the horizontal variable, in this case $z_{M,1,2}$, then $R_M(t)$ would have the slope $1/z'_{M,1,2} > 0$ [where the inequality follows from equation (3.3)], as shown in Figure 4.1. However, following (here) the economics convention earlier expressed, we are regarding the *vertical* variable z_2 as independent, for which the run of the corresponding slope is provided by a change in z_2. Thus, $z'_{M,1,2} > 0$ itself correctly expresses the slope that we require.

In addition, we need a name for the value of $z_{1,2}$ at which R_M intercepts the party-1 partial reaction (the horizontal) axis. Let it be given by

$$\gamma_{M,1,2}(t) \equiv z_{1,2} \text{ -intercept of } R_M(t) \tag{5.1}$$

In the next few steps, for legibility I omit the t - argument. From the construction of the straight line R_M, the value of $z_{M,1,2}$ evidently is given, in terms of intercept and slope, by

$$z_{M,1,2} = \gamma_{M,1,2} + (z'_{M,1,2}) \cdot (z_2) \tag{5.2}$$

Combining (4.1), (4.2) and (5.2), we get

$$\begin{aligned} \dot{z}_{1,2} &= k_1\{z_{1,2} - z_{M,1,2}\} \\ &= k_1\{z_{1,2} - [\gamma_{M,1,2} + (z'_{M,1,2}) \cdot (z_2)]\} \\ &= -k_1(z'_{M,1,2}) \cdot (z_2) + k_1 z_{1,2} - k_1 \gamma_{M,1,2} \end{aligned} \tag{5.3}$$

where, for ease of comparison with equation (5.5) below, the final line changes the order in which the terms appear. Also, we can re-express the coefficients using the following substitutions, with the time argument restored,

$$\begin{aligned} a_{M,1,2}(t) &\equiv -k_1(t) z'_{M,1,2}(t) > 0 \\ b_1(t) &\equiv -k_1(t) > 0 \\ g_{M,1,2}(t) &\equiv -k_1(t)\gamma_{M,1,2}(t) \end{aligned} \tag{5.4}$$

with the aid of which (5.3) is written as

$$\dot{z}_{1,2}(t) = [a_{M,1,2}(t)] \cdot [z_2(t)] - b_1(t) z_{1,2}(t) + g_{M,1,2}(t) \ .$$

At this juncture we make two observations about the meaning of the above. First, as previously mentioned, while the discussion has referred to parties 1 and 2, it could just as well refer to any two parties i and j. So the above equation can be written more generally as

$$\dot{z}_{ij}(t) = [a_{M,ij}(t)] \cdot [z_j(t)] - b_i(t) z_{ij}(t) + g_{M,ij}(t) \ , \ i \neq j$$

(5.5),

where the z -values are given by the corresponding generalization of equations (3.1), as

$$\begin{aligned} z_{ij}(t) &\equiv \ln[r_{ij}(t)] \\ z_k(t) &\equiv \ln[r_k(t)] \end{aligned}$$

(5.6),

i, j, k referring to any of the N parties in the system.

Again, for the next few lines we omit the time argument. The second observation is that, in the above, $a_{M,ij}$ and $g_{M,ij}$ have so far not received any substantive meaning; they are just uninterpreted quantities that arise in the course of developing the utilities and dynamics of provocation and partial reaction. So, there is no inconsistency if we now give them substantive interpretation. Suppose we interpret them to be the reaction and grievance coefficients, respectively, in Richardson's (1960) arms race equations. Then, from the above, it is evident that b_i equals the fatigue coefficient in those equations. So far as the (partial) response of party i to party j is concerned, equations (5.5) are, thus, the well-studied arms race or "Richardson process" equations (to use Boulding's term). Note the latter term contemplates the possibility that the process being described applies to something other than armaments. The above derivation for a system of two parties is exactly the one provided by Boulding in the above cited source, except that here the grievance, reaction, and fatigue coefficients are regarded to be time varying.

From equation (5.5) we conclude that

$$\sum_{j=1}^{N} \dot{z}_{ij} = \sum_{j=1}^{N} a_{M,ij} z_j - b_i \sum_{j=1}^{N} z_{ij} + \sum_{j=1}^{N} g_{M,ij} \ , \ i \neq j \ ,$$

(5.7)

where again the argument t is omitted. The connection from the above to the total response, of party i to all other parties, is now provided by the composition assumption, equation (1.1), from which we see that

$$\ln r_i = \sum_{j \neq i, \, j=1}^{N} \ln r_{ij} \ .$$

Using equations (5.6) the above can be written

$$z_i = \sum_{j \neq i, \, j=1}^{N} z_{ij}$$

(5.8),

from which we see that

$$\dot{z}_i = \sum_{j \neq i, \, j=1}^{N} \dot{z}_{ij}$$

(5.9).

With the aid of (5.8) and (5.9), equation (5.7) can be written

$$\dot{z}_i = \sum_{j=1}^{N} a_{M,ij} z_j - b_i z_i + \sum_{j=1}^{N} g_{M,ij} \ , \ i \neq j$$

which, if we abbreviate the final term on the right to $g_{M,i} \equiv \sum_{j=1}^{N} g_{M,ij}$, becomes

$$\dot{z}_i(t) = \sum_{j=1}^{N} [a_{M,ij}(t)] \cdot [z_j(t)] - b_i(t) z_i(t) + g_{M,i}(t) \ , \ i \neq j \ , \tag{5.10}$$

(5.10),

with time arguments restored. This is a generalization of the N -many parties formula originally suggested by Richardson (1960) because $a_{M,ij}$, b_i , and $g_{M,i}$ are variable and depend, via equations (5.4), on the slope and intercept value of the straight line tangent to the reaction curves at the various points M (one for each other party j to which party i is reacting) introduced in Part 2, above. For straight line reaction curves, however, the slopes and intercepts will be constant for each other party j which, as Boulding points out, recovers Richardson's original linear differential arms race equations. In that case the (all now constant) $a_{M,ij}$, b_i , and $g_{M,i}$ become the reaction, fatigue, and grievance coefficients, respectively, of Richardson's model for N -many parties. As previously suggested in Part 4, above, generalizing from constant to time varying coefficients is central in what follows.

Two further points to observe concern, first, the signs of the reaction and fatigue coefficients a and b , respectively, and, secondly, the grievance term g in equations (5.4) and (5.10). Turning first to the coefficients, these are conventionally positive: in the case of the former, this reflects the idea that the provocations of other parties increases the rate of change of the referent party provocation and, in the case of the latter, it reflects the idea that a greater degree of effort retards one's own rate of change. If these signs are allowed to be negative then the system may become unstable, corresponding to a run-away arms race in the original interpretation. [This stability issue was explored by Richardson (1960).] For constant coefficients such an instability would put in question the realism of the equations; a real system could not perpetually exhibit the exponential growth implied by, for instance, a negative b -value in equation (5.10). For varying coefficients the situation would be less clear-cut; one might realistically imagine them varying in such a way as to reflect a temporary instability followed by a stabilizing change in coefficient values; the time variation expressed in (5.10) anticipates this possibility. [Note the empirical evidence for $-b > 0$ in the case of the United States during its competitive period with the Soviet Union (Wagner, et al., 1974).]

Later it becomes clear [Part 12, discussion leading to equation (12.23)] that the fatigue coefficient cannot assume a zero-value, thus it cannot change sign while continuously varying; thus we continue to assume this coefficient to be positive. There is, however, no corresponding restriction on the reaction coefficients. A negative a -value could mean that the referent party finds the "provocation" of the other party to be a source of security, as in the case of a nation finding protection in a powerful ally. (This would fit the recurring complaint alleged during an earlier time, about allies relying on US armaments to get by with fewer of their own.) Likewise, a vanishing reaction coefficient would correspond to (a reactive) indifference toward the other party. In terms of the partial response utilities discussed in Part 2, these variant or "abnormal" cases would correspond to the local maxima of the utility curves respectively aligned from upper left to lower right, or aligned vertically, one above the other. Thus the following assumes no restriction on the sign of the reaction coefficients.

Similar reasoning allows negative and zero values for the grievance term g ; that means, of course, that negative "grievance" (altruism?) is also possible toward other parties; so also is (a non-reactive) indifference. In view of this generalized meaning an important additional point reflects its status, now generalized also as a varying function of time. The consequence is that one is free to define the value of g as equal to some other function of time; there is no contradiction in this because nothing else is being said about the term. We will use that freedom in Part 12. (Of course logical consistency is not the same as evidence; the grievance values could still be false given some empirical interpretation.)

An implication of this freedom, in the values assigned to reaction coefficients and grievance terms, is that *any two parties* (units, nations, etc.) represented in the model can have a mutual relationship defined by equations (5.10); it is not necessary that they be in an adversarial or competitive relationship as conventionally understood. Moreover, their variability allows dyadic (two party) relationships to change: competition can appear then, after a time, disappear.

6. Richardson Process as the Cumulative Effect of Forgetting.

A different but, as it turns out, equivalent point of view is suggested by Abelson (1963); however, to use his idea we must modify it in ways I will discuss in this part and in Part 7. Let us begin with what Abelson did. For simplicity we again speak of two parties 1 and 2, each of which is provoking the other. These provocations are remembered but, as time passes, this memory becomes dimmer; gradually, the parties forget any specific incident in the past. At any given moment the provocation sent by each party is proportional to its accumulated memory of all past provocations received. These ideas can be expressed as follows. Let

$M_1(t,\tau) \equiv$ party 1 memory at time $t \geq \tau$, of provocation received by it from party 2 at time τ ,

$A_2(\tau) \equiv$ provocation received by party 1 from party 2 at time τ ,

$A_1(t) \equiv$ partial response of party 1 to party 2 at time t .

$\gamma_1(t)$, $\lambda_1(t)$, and $\theta_{1,2}(t)$ are time varying coefficients specific to party 1 or both of them, respectively,

$K \equiv$ a constant.

(6.1).

Unlike Abelson's original, the quantities $\lambda_1(t)$ and $\theta_{1,2}(t)$ are time varying in the above; $\gamma_1(t)$ and K are entirely new. Also, I have added the notation $M_1(t,\tau)$, so that Abelson's original expression [his equation (5)] can be broken into the two equations (6.2) and (6.3), below. In what follows, the expression $A_2(t)$ in (6.4) has the analogous meaning, at time t for party 2, as the corresponding symbol with subscript 1 has for party 1; that is, it is the partial response of party 2 to party 1 at time t. In application, the expression "partial response" will reflect the idea presented earlier, that there may be multiple parties and that the total response by a given party is the combination of its separate responses to each of the others, as given in equations (1.1) and (5.8). The various quantities are assumed to be differentiable with respect to time.

The accumulation assumption is then expressed by

$$A_1(t) = [\int_a^t M_1(t,\tau)d\tau] + \gamma_1(t) + K$$

(6.2),

and the diminution of memory assumption, by

$$M_1(t,\tau) = A_2(\tau)\theta_{1,2}(\tau)e^{-\lambda_1[t-\tau]} \ , \ \lambda_1 > 0$$

(6.3).

Figure 6.1 shows an example of equation (6.3). In equation (6.2), to what Abelson presents I have appended the second and third terms. The second term, the quantity γ_1 , furnishes a "baseline" rate of change in $A_1(t)$. It leads to the grievance term, as described later; similarly, $\lambda_1(t)$ and $\theta_{1,2}(t)$ lead to the fatigue and reaction coefficients. The constant K is included in equation (6.2) to give the expression a more general form. In effect, it is reflecting that there is no intrinsic zero-point to the value of $A_1(t)$. I have also changed the lower limit of integration from Abelson's 0 to the quantity $a < t$. [One might ask, How far back in time should one begin the process of integration in (6.2)? It turns out not to matter; any constant value of a will work. However, for later uses (not yet encountered), one might want to be assured that provocations previous to the specific chosen a are sufficiently remote to be negligible. The condition making that choice of a possible is that the integral be convergent as $a \to -\infty$. Alternatively, K can be regarded as the value of the integral of $M_1(t,\tau)$ from $-\infty$ to a , whatever the latter value may be.]

In Part 7, I show that, by differentiating both sides of equation (6.2) with respect to time t and taking into account the specific value of $M_1(t,\tau)$ given by (6.3), one obtains as a special case

$$\dot{A}_1(t) = -\lambda_1 A_1(t) + \theta_{1,2}(t)A_2(t) + \lambda_1[\gamma_1(t) + K] + \dot{\gamma}_1(t)$$, (not sufficiently general)

(6.4).

The calculation leading to (6.4) is developed in the previously cited Abelson (1963), except that (a) Abelson assumes the coefficient θ_1 to be constant, and (b) Abelson does not include the two final terms. In the Abelson original and in the special form (6.4), λ_1 is regarded as constant; thus the omission of the argument t . The constant K in (6.2) disappears upon differentiation. (Further assuming γ , $\dot{\gamma}$, and $K = 0$ gets the original Abelson expression, which did not have a grievance term.)

Not withstanding the indicated changes, the derivation remains valid. However as the warning indicates, (6.4) is not general enough to avoid two assumptions that are invalid, given the intended use to follow. In brief, these invalid assumptions are: the fatigue coefficient λ is a constant and there is no time delay between provocation by one party and response by another. In Part 7 we turn to the necessary generalization of (6.4), so as to avoid making these assumptions.

7. Forgetting with Delay

We now introduce the idea that there may be some delay in the process, described above, that begins with initial recognition of / response to a provocation, then continues with its forgetting. Where might this delay of initial recognition originate? One may suppose that, realistically, there is some degree of "psychological," "bureaucratic" or other decision making organizational or computational process-time constraint, or some combination of them. Every known (or conceived) biological, chemical, or nervous system, and every artificial computing system would seem to be characterized by greatly varying but non-zero values of such time constraints because "recognition / response" or "computation" by a physically real decision making system necessarily entails transmission of signals within it.

[Further, even without naming more specific constraints, because all physically real systems have non-zero spatial dimensions we know that the speed of light thus imposes an upper bound to the speed of signal transmission; this constraint is *physical, not an analogue*. Accordingly, computer CPU designers try to make their devices as small as possible. On human time and distance scales the relevant dimensions, even of biological systems, appear small; the straight line distance from one of my ears to my mouth is a very tiny fraction of a light-second. But consider the non-straight line path of an input signal that must bounce around quite a bit from one neuron to another– millions of them –before it and other inputs are processed into an output (perhaps in the form of something I say, in response to something I heard). That could be a very long path; maybe on the order of the number of light seconds corresponding to the time it takes to read this sentence. (It just took me about 5 seconds; 5 light seconds $\cong$ 1.5 million kilometers. This forms an upper bound to the actual path length.)]

Let the amount of this Abelson process delay be denoted by $\varepsilon(t)$, which is written as a function of t to recognize the possibility that the delay varies with time. In what follows, let us assume that this is a continuous, differentiable function. An illustration of the Abelson representation with delay is given in the right-hand plot of Figure 6.1, which shows the (right-) shifted decay function corresponding to $\varepsilon(t) > 0$. To distinguish the degree of delay, we adopt new notation $z_{1,2}[t, \varepsilon_{1,2}(t)]$ for the amount of partial response of side 1 to side 2, where the first argument gives, as before, the referent ("present") time and the second argument gives the amount of delay in recognition, by side 1, of the initial provocation by side 2. That $\varepsilon_{1,2}(t)$ has subscripts denoting the two parties, reflects the idea that the delay may be specific to the particular pair of parties. This anticipates the application, below.

Now we are ready to backtrack to the part of the discussion beginning with Abelson's original idea, so as to introduce this new delay quantity $\varepsilon_{1,2}(t)$. For convenience we also define

$$u_{1,2}(t) \equiv t - \varepsilon_{1,2}(t). \tag{7.1}$$

Using the notation of (6.1), the previous diminution of memory assumption (6.3) becomes

$$M_1(t,\tau) = A_2(\tau)\theta_{1,2}(\tau)e^{-\lambda(t)[t-\varepsilon(t)-\tau]} \text{ , } \lambda_1(t) > 0 \text{ ,} \qquad (7.2).$$

where, to improve legibility in the exponent, I omit the subscripts 1, 2 denoting the two parties. Note that, since $t - \varepsilon(t) - \tau = t - [\varepsilon(t) + \tau]$, one way to interpret the exponent in (7.2) is that, consistent with earlier discussion of a delay in recognition, response to the provocation is as though it had occurred at the later time $\tau + \varepsilon(t)$.

The previous accumulation assumption (6.2) becomes

$$A_1(t) = [\int_a^{t-\varepsilon(t)} M_1(t,\tau)d\tau] + \gamma_1(t) + K$$
$$= [\int_a^{u} M_1(t,\tau)d\tau] + \gamma_1(t) + K \ .$$

(7.3)

Using (7.1) the operator differentiating with respect to time becomes

$$\frac{d}{dt} = \frac{d}{du} \cdot \frac{du}{dt} = \frac{du}{dt} \cdot \frac{d}{du} = \frac{d}{dt}[t - \varepsilon(t)]\frac{d}{du} = [1 - \dot{\varepsilon}(t)]\frac{d}{du} \ .$$

(7.4)

To establish a point used shortly, equation (7.1) presents $u = f(t)$ as a function of t . One might ask whether the inverse function $t = f^{-1}(u)$ is well defined. This turns out to be the case, based on reasoning developed starting in Part 8, below [where the quantities presently under discussion (i.e. in this Part 7), are related to a geometrical construct, the "social space-time reference frame" $\overline{S}$]. As developed starting there, one finds [equation (11.16)] that $\dot{\varepsilon} < 1$. Equation (7.4) shows that $du/dt = 1 - \dot{\varepsilon}(t)$. From this and equation (11.16) we find $du/dt > 0$. This shows that u is a strictly monotonic function of t , thus the inverse function $t = f^{-1}(u)$ is well defined. Thus any function $g(t)$ can also be written as a function of u : $g[f^{-1}(u)] \equiv \overline{g}(u)$.

To incorporate the new delay element, and using equation (7.3), we find

$$\dot{A}_1(t) = \frac{d}{dt}\left[\int_a^u M_1(t,\tau)d\tau\right] + \dot{\gamma}_1(t) \, ,$$

(7.5)

where again K disappears because it is constant. Continuing with the first term on the right

$$\frac{d}{dt}\int_a^u M_1(t,\tau)d\tau$$

$$= \frac{d}{dt}\int_a^u A_2(\tau)\theta_{1,2}(\tau)\exp\{-\lambda_1(t)[t - \varepsilon_{1,2}(t) - \tau]\}d\tau$$

the above from (7.2),

$$= \frac{d}{dt}\int_a^u A_2(\tau)\theta_{1,2}(\tau)\exp\{-\lambda_1(t)[t-\varepsilon_{1,2}(t)]\}\cdot\exp\{-\lambda_1(t)[-\tau]\}d\tau$$

$$= [\int_a^u A_2(\tau)\theta_{1,2}(\tau)\cdot\exp\{-\lambda_1(t)[-\tau]\}d\tau]\frac{d}{dt}\exp\{-\lambda_1(t)[t-\varepsilon_{1,2}(t)]\}$$
$$+\exp\{-\lambda_1(t)[t-\varepsilon_{1,2}(t)]\}\frac{d}{dt}\int_a^u A_2(\tau)\theta_{1,2}(\tau)\cdot\exp\{-\lambda_1(t)[-\tau]\}d\tau$$

$$= [\int_a^u A_2(\tau)\theta_{1,2}(\tau)\cdot\exp\{-\lambda_1(t)[-\tau]\}d\tau]\exp\{-\lambda_1(t)[t-\varepsilon_{1,2}(t)]\}\frac{d}{dt}\{-\lambda_1(t)[t-\varepsilon_{1,2}(t)]\}$$
$$+\exp\{-\lambda_1(u)[t-\varepsilon_{1,2}(t)]\}[1-\dot{\varepsilon}_{1,2}(t)]\frac{d}{du}\int_a^u A_2(\tau)\theta_{1,2}(\tau)\cdot\exp\{-\lambda_1(u)[-\tau]\}d\tau \text{ ,}$$

the above using (7.4) and the result following it, that t is a function of u (the latter so as to obtain the change from $\lambda_1(t)$ to $\lambda_1(u)$ in the second term),

$$= [\int_a^u A_2(\tau)\theta_{1,2}(\tau)\cdot\exp\{-\lambda_1(t)[-\tau]\}d\tau]\{-\lambda_1(t)[1-\dot{\varepsilon}_{1,2}(t)]-\dot{\lambda}(t)[t-\varepsilon(t)]\}\exp\{-\lambda_1(t)[t-\varepsilon_{1,2}(t)]\}$$

$$+\exp\{-\lambda_1(u)[t-\varepsilon_{1,2}(t)]\}[1-\dot{\varepsilon}_{1,2}(t)]\cdot A_2(u)\theta_{1,2}(u)\cdot\exp\{-\lambda_1(u)[-u]\} \text{ ,}$$

where the affect of d/du on the integral to its right uses a generalization of the “replacement operator” aspect of the fundamental theorem of integral calculus,

$$= \{-\lambda_1(t)[1-\dot{\varepsilon}_{1,2}(t)]-\dot{\lambda}(t)[t-\varepsilon(t)]\}\int_a^u A_2(\tau)\theta_{1,2}(\tau)\cdot\exp\{-\lambda_1(t)[-\tau]\}\cdot\exp\{-\lambda_1(t)[t-\varepsilon_{1,2}(t)]\}d\tau$$

$$+\exp\{-\lambda_1(u)\cdot u\}[1-\dot{\varepsilon}_{1,2}(t)]\cdot A_2(u)\theta_{1,2}(u)\cdot\exp\{\lambda_1(u)\cdot u\} \text{ ,}$$

the above using (7.1) to replace $t-\varepsilon_{1,2}(t)$ with $u \equiv u_{1,2}(t)$ in thesecond term,

$$= \{-\lambda_1[1-\dot{\varepsilon}_{1,2}(t)]-\dot{\lambda}_1[t-\varepsilon_{1,2}(t)]\}\int_a^u M_1(t,\tau)d\tau + [1-\dot{\varepsilon}_{1,2}(t)]\cdot\theta_{1,2}(u)A_2(u)$$

the above step using (7.2); and using (7.3) we find the above

$$= \{-\lambda_1[1-\dot{\varepsilon}_{1,2}(t)]-\dot{\lambda}_1[t-\varepsilon_{1,2}(t)]\}[A_1(t)-\gamma_1(t)-K]$$
$$+[1-\dot{\varepsilon}_{1,2}(t)]\theta_{1,2}(u)A_2(u)$$

$$
\begin{aligned}
&= -\{\lambda_1[1-\dot{\varepsilon}_{1,2}(t)] + \dot{\lambda}_1[t-\varepsilon_{1,2}(t)]\}A_1(t) \\
&\quad + [1-\dot{\varepsilon}_{1,2}(t)]\theta_{1,2}(u)A_2(u) \\
&\quad + \{\lambda_1[1-\dot{\varepsilon}_{1,2}(t)] + \dot{\lambda}_1[t-\varepsilon_{1,2}(t)]\}[\gamma_1(t)+K]
\end{aligned}
$$

(7.6).

The replacement operator generalization to which reference is made in the above consists of an integrand in which the upper bound of integration appears as an argument; namely in $\lambda_1(u)$. Finally, equations (7.5) and (7.6) together show that the partial response of party 1 to party 2 is given by

$$
\begin{aligned}
\dot{A}_1(t) &= -\{\lambda_1[1-\dot{\varepsilon}_{1,2}(t)] + \dot{\lambda}_1[t-\varepsilon_{1,2}(t)]\}A_1(t) \\
&\quad + [1-\dot{\varepsilon}_{1,2}(t)]\theta_{1,2}(u)A_2(u) \\
&\quad + \{\lambda_1(t)[1-\dot{\varepsilon}_{1,2}(t)] + \dot{\lambda}_1(t)[t-\varepsilon_{1,2}(t)]\}[\gamma_1(t)+K] + \dot{\gamma}_1(t)
\end{aligned}
$$

(7.7).

Using (7.1) and (7.4) this can also be written

$$
\begin{aligned}
\dot{A}_1 &= -(\lambda_1\dot{u}_{1,2} + \dot{\lambda}_1 u_{1,2})A_1 + \dot{u}_{1,2}\theta_{1,2}A_2 + (\lambda_1\dot{u}_{1,2} + \dot{\lambda}_1 u_{1,2})(\gamma_1 + K) + \dot{\gamma}_1 \\
&= -\lambda_1\dot{u}_{1,2}A_1 + \dot{u}_{1,2}\theta_{1,2}A_2 + \lambda_1\dot{u}_{1,2}(\gamma_1 + K) + \dot{\gamma}_1 + \dot{\lambda}_1 u_{1,2}(\gamma_1 + K - A_1) \ .
\end{aligned}
$$

(7.7a)

Note that the final term on the right makes $\dot{A}_1$ dependent on the value of $u = t - \varepsilon$. It might be argued that this is an unwanted feature since it says that the impact of changes in λ is proportional to the value of the time, whereas the latter is an artifact purely of the choice of a time origin. If so, one can introduce the added assumption that

$$\gamma_i + K = A_i$$

(7.7a)'

(γ_i linear in the response A_i), by which the term disappears.

Using the previous more general index notation i and j , equation (7.7a) can be written

$$\dot{A}_i = -\lambda_i\dot{u}_{ij}A_i + \dot{u}_{ij}\theta_{ij}A_j + \lambda_i\dot{u}_{ij}(\gamma_i + K) + \dot{\gamma}_i + \dot{\lambda}_i u_{i,j}(\gamma_i + K - A_i) \ .$$

(7.7b)

To connect the modified Abelson approach to prior discussion, note that, from their positions in equation (7.7b), the quantity multiplying A_i in the first term, the quantity multiplying A_j in the second term, and the entire third and fourth terms taken together, respectively have the same positions as fatigue coefficient, reaction coefficient, and grievance term, in that form of Richardson's equations expressed in equation (5.5). Thus the Abelson approach gives an entirely equivalent approach to those of Boulding and of Richardson, as modified above, provided we make the following identifications:

$$\begin{array}{rcl} A_i & \leftrightarrow & z_{ij} \\ A_j & \leftrightarrow & z_j \\ \lambda_i(1-\dot{\varepsilon}_{ij}) = \dot{u}_{ij}\lambda_i & \leftrightarrow & b_i \\ (1-\dot{\varepsilon}_{ij})\theta_{ij} = \dot{u}_{ij}\theta_{ij} & \leftrightarrow & a_{M,ij} \\ \lambda_i\dot{u}_{ij}(\gamma_i+K)+\dot{\gamma}_i+\dot{\lambda}_i u_{i,j}(\gamma_i+K-A_i) & \leftrightarrow & g_{M,ij} \end{array} \qquad (7.8).$$

Since no empirical definition has yet been made of the Abelson coefficients, we are free to make the indicated identifications. Thereby a new set of ideas about arms racing is suggested, from the mathematical viewpoint entirely compatible with the ones already expressed. After the further development in the next numbered part, we shall continue to use the previously introduced notation, appearing on the right hand side of (7.8), to refer to the elements of a Richardson process, which now includes the entirely consistent additional meaning—*as a process that results from the accumulation of past provocations and subsequent forgetting*—identified by Abelson. Note that the quantities z_{ij} and z_j in (7.8) are defined with respect to each of the arguments t, τ, and u .

Now we can imagine a special case where the fatigue λ_i and delay $\varepsilon_{ij}(t)$ are constant; then $\dot{\varepsilon}=0$, $\dot{u}=1$, and (7.7) becomes simplified. This case yields the form of the previous Abelson equation (6.4) (with indices generalized to i and j):

$$\dot{A}_i(t) = -\lambda_i A_i(t) + \theta_{ij}(t)A_j(t) + \lambda_i[\gamma_i(t)+K] + \dot{\gamma}_i(t) \text{ , (not sufficiently general).}$$

We will see, however (Part 12), that λ_i cannot be constant when the Abelson reasoning is put in the geometric context that we begin to develop in Part 8; on that account the above is insufficiently general.

From equations (7.7b) and (7.8) we find

$$\dot{z}_{ij} = [a'_{M,ij}(u_{ij})]\cdot[z_j(u_{ij})] - b'_{ij}z_{ij} + g_{M,ij} \text{ , } i \neq j \text{ ,}$$

where

$$u_{ij} \equiv t - \varepsilon_{ij}(t) \text{ ,}$$
$$a'_{M,ij}(u_{ij}) \equiv \dot{u}_{ij}\cdot a_{M,ij}(u_{ij}) \text{ ,}$$
$$b'_{ij} \equiv \dot{u}_{ij}\cdot b_i \text{ ,} \qquad (7.9)$$

and $\varepsilon_{ij}(t)$ is the Abelson process delay at time t of the recognition by party i of the provocation by party j .

In the above all quantities are functions of the argument t . For visual simplicity these arguments are omitted but the intermediate variable u_{ij} is retained to emphasize that the corresponding quantities are retarded (time delayed) and party j - specific. I take equation (7.9) as the *generalized Richardson process partial reaction equation incorporating the things we have just built into it: variable reaction and fatigue coefficients, grievances, and recognition delay times*. To connect with the previous discussion, the partial response equation (5.5) is a special case: For $i = 1$, $j = 2$ (i.e. for the response of party 1 to provocation by party 2), and for $\varepsilon, \dot{\varepsilon} = 0$, equation (7.9) becomes the original (5.5).

Finally, emulating equation (5.10), we can sum across the value of j in equation (7.9) to obtain the total response of party 1 to all others:

$$\dot{z}_i = \sum_{j=1}^{N} [a'_{M,ij}(u_{ij})] \cdot [z_j(u_{ij})] - b_i z'_i + g_{M,i} \ ,$$

where

$$\dot{z}_i \equiv \sum_{j=1}^{N} \dot{z}_{ij}(t, \varepsilon_{ij}) \ ,$$

$$z'_i \equiv \sum_{j=1}^{N} \dot{u}_{ij} \cdot z_{ij} \ ,$$

$$g_{M,i} \equiv \sum_{j=1}^{N} g_{M,ij} \ ,$$

and $i \neq j$.

(7.10)

Note that the original simple summation of partial reactions, forming the total reaction z_i of party i , has been replaced by the weighted sum z'_i ; upon assuming that the delay times are constant one recovers the original total reaction in the fatigue term. (In Part 12 we will see how it is possible always to recover the role of the original z_i in the fatigue term, by redefining the grievance term to include part of z'_i .)

In effect, the above regards that the effect of every provocation in a Richardson process is shifted *forward* by some varying amount of time. For example, suppose the referent present time is $t = 1990$ (years) and the delay is $\varepsilon_{1,2}(1990) = 10$ years. Then equation (7.9) says that the relevant provocation of party 1 by party 2 at the "present time" 1990 is given by $z_2(u_{1,2}) =$ $z_2[t - \varepsilon_{1,2}(t)] =$ $z_2(1990 - 10) =$ $z_{1,2}(1980)$; that is, the 1990 value is what would have been observed at the year 1980 had there been no delay. Thus the result is a forward shift in the effect of the provocation, from 1980 to 1990. In addition, for time-*varying* delay, the reaction and fatigue coefficients are altered by the factor $1 - \dot{\varepsilon}_{1,2}(t) = \dot{u}_{i,j}(t)$ appearing in equation (7.7).

To summarize thus far, to Richardson's original idea we have added perspectives provided by Boulding, McGuire, and Abelson; and we have added the ideas that certain of the parameters may vary over time and that there will be a time delay in the partial reaction by one party to the provocations of another. The parameters to the Richardson process, as thus amended, have the meanings shown in Table 7.1.

Table 7.1. Equivalent parameter interpretations in Richardson process, with amendments. [See equation (7.10). All parameters are time-varying.]

Parameter:	Richardson:	Boulding – McGuire:	Abelson:
b_i	fatigue	rate of convergence	rate of forgetting
$a'_{M,ij}(u_{ij})$	reaction	$\leftrightarrow$ maximal utility for given provocation	reaction at prior times $\tau < u_{ij} = t - \varepsilon_{ij}(t)$
$g_{M,i}$	grievance	$\leftrightarrow$ nominal reference level of utility	baseline partial response change rate

Note that each of the three parameters *can consistently receive all three of the interpretations available to it*. This is because no operational empirical interpretation has yet been offered to any of them. Upon assignment of such an interpretation—say, a definite procedure to empirically estimate the Richardson fatigue coefficient—then either of two alternatives might happen with the remaining two quantities: the fatigue coefficient could also become the definition of rate of convergence and rate of forgetting; or, independent definitions might be assigned to rates of convergence or forgetting (or to both). In the latter alternative, the assertion, in the table, of the equivalence of the forms given, would amount to an empirically falsifiable proposition. The same applies to the other two parameters. In each instance, however, the former possibility, that of tautological equivalence among the different interpretations, should also be kept in mind.

I take the point of view that it is better *not* to make such choices at this juncture; better, is to wait until the system is built to as far as we know how at this point. By assigning empirical meanings at that later juncture, we will be populating, simultaneously, a large number of empirical assignments, the better to see whether, in concert, they accomplish what our intuition says we should want.

8. Social Field-Theoretic and Space-Time Perspective.

Now turning to the second key element of the discussion, we begin with the social field-theoretic perspective suggested by Wright (1955) and elaborated by Rummel (1965), concerning social units—which Wright (this and references following in Wright, 1961), p.401, characterized as "each...'system of action' or 'decision maker' "—entities which, in the above we have called the "parties", and of which nation states are prominent examples. In essence, Wright appears to regard that such entities are particle-like objects. Such entities would have coordinates by which they would be located in a multi-dimensional "social field." (Though he does not use that word, the little circles depicting nations, in the figures on page 404, and the reference to "coordinates" bespeak a particle conception.) The coordinates for each party would be derived from various of its characteristics; exactly which Wright deferred choosing, with the remark (p.401) that "Much experiment would be necessary to decide what co-ordinates could most usefully be employed." Over time, spatial movements by each party would occur; Wright (pp.403-404) suggests some illustrative possibilities.

Wright's discussion largely consists of nuanced considerations concerning how various factors, characterizing the parties and commonly regarded as affecting global society, might be translated into various dimensions locating the parties in the social field. The representation of behavior *between* nations receives less attention in this particular source. One suggestion (p.403) is that "Persistent factors operating among states and influencing their capabilities may be conceived in terms of different aspects of distance between them." This suggestion receives greater attention by Rummel (1965), who proposes that the behavior between parties be interpreted as *forces* between them.

From these social field-theoretic ideas we now take the part that will form a framework to be synthesized with the Richardson process framework and also will serve other purposes. First, it seems clear that Wright and Rummel were thinking of their social field as residing in an abstract geometric space, within which the various parties would be positioned, and which positions would be represented by their coordinates; and changes of position, by changes in the coordinates. With some added interpretation, the suggestions about persistent factors, forces, and mutual distances of the preceding paragraph can be worded as follows: the "persistent factors" give rise to forces that alter the relative motions of the parties; these alterations in turn affect the relative positions of the parties and, thus, the coordinates that translate into the distances between the parties. Thus interpreted and modified, we will keep those ideas of "space" and of what goes in it, and with it. We will also keep the idea that parties may exert "forces" on each other; and that their motions may be affected by those forces.

Upon reading Wright and Rummel it is easy to think of physics, as apparently they did, since the societal concepts just named correspond to similar (or similarly named) physical counterparts which therefore appear to serve as exemplars. The following will carry considerably further this practice of emulating physical concepts. We will discover that these additional concepts connect with the Richardson process constructs already named in Table 7.1, and with other world political and societal concepts as well. One other point should be made clear: just as for Wright and Rummel, it is the form of the mathematics that is to be borrowed, not the content; this is *not* an application of physics in the sense of a physical consideration of social phenomena, interesting and important though the latter may be (and undoubtedly is). In this part we begin this emulation of mathematical physical concepts.

Central to our emulation is the physical picture of geometry. In that picture, geometry is not just of "space" as we conventionally think of it, but also of time. Is this space-time analogy actually needed? In Part 18 we will see that it is. We call this geometric conception the *social space-time*, and introduce it in the form of a social space-time diagram, the pair of coordinate axes shown as Figure 8.1, and develop it further, below.

In general we assume the social space-time to be characterized by one social time dimension, with corresponding coordinate x^0 and coordinate axis X^0, and several social spatial dimensions, with corresponding coordinates x^k and coordinate axes X^k, $k = 1,2,...N$, where the superscripts ^{0} and ^{k} are dimensional indices (not exponents). In addition we will conventionally assume $N = 3$. Why choose that particular value of N ? Primarily we choose it out of convenience: the mathematical physics uses that value, so it is a mathematically well understood place to start. Also there are substantive considerations but they are not well understood at this juncture. [One of them is fragmentary evidence that 3 dimensions are sufficient to accommodate the referent "linkage" data as discussed Part 13, below (Williamson 1985).] We also assume that the social space-time forms a continuum, roughly meaning that it is possible to connect any two points in the space-time with a shortest or longest line, all the points on which are also in the space-time. Further, in this social space-time continuum angles between directed line segments are well-defined quantities.

How shall we position the parties in this continuum? Abstractly, we do this by imagining that we have chosen space coordinates for each party at each moment. Following custom, we will call each complete collection of such coordinates, naming exactly once all time-location combinations, a reference frame. This means naming in principle, not necessarily explicitly or empirically; often a frame will be designated simply by referring to its coordinate axes. In such a reference frame, each combination of space and time coordinates represents an event: namely, something happening at a particular place, as indicated by its spatial coordinates, and at a particular time, as indicated by its time coordinate. (By “complete” collection of coordinates I mean in the algebraic sense of a complete set of vectors spanning a vector space.)

Returning to Figure 8.1, for simplicity one social spatial coordinate, x^1 , is depicted and associated with the horizontal axis X^1, together with a social time coordinate, x^0 , associated with the vertical axis X^0. Each point in this figure represents an event, though only two of the coordinates, x^0 and x^1, are shown. The lines corresponding to the coordinate axes can be thought of as directed line segments of infinite length in both directions. As the figure suggests, these axes may be considered to be mutually perpendicular. Unless otherwise stated, for any set of axes this perpendicular property will be assumed in what follows.

Also shown in Figure 8.1 are two 45-degree lines. In the physical exemplar these lines depict the paths of light rays converging from below and toward the origin, marked O in the diagram (as in this picture; or, in other pictures, diverging above and away from the origin; or both). In the social emulation we do not have light rays but do have *signals* that represent the transmission of a form of societal information (just as light transmits information in the physical exemplar). In the figure, the 45-degree lines represent the paths of such signals. Note that these signals are *not* the customary physical communications—at least as we normally think of them—that occur when one person engages in face-to-face conversation with another, picks up the telephone to talk with another, sends a message on the internet, or travels from one place to another; rather, the speed of transmission of societal space-time signals may be much slower than the fastest forms of physical communication. In Part 9 we will relate societal signal speed to the previously discussed Richardson process response delay.

As in the physical exemplar, we assume that the rate of transmission of these societal signals is constant—everywhere showing movement through the same amount and in the same direction of social space during a unit amount of elapsed social time. That ratio, of social-spatial movement to social-time movement, appears as the slope of the signal lines in the figure. Since rate of transmission is constant, that ratio and its corresponding slope must be constant, thus the signals must appear as straight lines in a space-time diagram, as they do in Figure 8.1. We also will make the very critical assumption that this rate of signal transmission is the *same* as viewed in every reference frame, an assumption to which we return in Part 10, below. As will be seen, space distances in the continuum are defined in such a way as to make this assumption logically true in one class of reference frames, of a type called $\bar{S}$, discussed below. The assumption will apply in all other reference frames.

In what follows I will use the letter c to denote this fundamental speed of signal transmission in social space-time. As the reader may know, in its physical usage c symbolizes the speed of light. Here, and often in what follows, I will use the same notation as customarily is used for the physical quantity being emulated. For much of the discussion we could just as well and, except where noted, we will, assume units of social time and social space measurement such that

$$c = 1 \ . \tag{8.1}$$

For clarity, however, I will often continue to include the symbol c in the relevant expressions. In addition, it is important to note that this societal signal speed parameter is dimensionless in our usage of it. This arises because (in contrast to the physical exemplar, where time and space units are conventionally distinguished) the measurement scheme, to be introduced below, uses time units to define both time and space distances; thus if, for example, the year is adopted as the time unit, then this unit, when applied to c , which is distance divided by time, becomes the dimensionless $[\text{year}]/[\text{year}]=1$.

For simplicity and where no confusion would arise, in speaking of social space and social time I will usually omit the “social” qualifier. One of the contexts in which it *will* be essential to distinguish “social” time is when we are also talking about changes over physical time, t . Moreover, while those two senses of time are distinct, they are connected. We need to consider this connection, as given in the following remarks.

To avoid confusion it may be helpful to preface the first remark by noting that, according to relativity physics, there are arbitrarily many differing time coordinates t , t' , t'' , t''' , ... respectively associated with observers that have differing relative motions in space. With few exceptions, however, for human-scale events here on Earth the above motion-related differences are negligible. (We disregard entirely gravitation-related time differences, since this discussion is based only on the analogy to special relativity.) In addition, there are differences of time measurement that can come about because of differing measurement units (seconds, hours, years, etc.) or origin (choice of the moment to be assigned the time value 0) . In what follows, let us assume that these differences have been eliminated by making a standard choice of unit and origin.

Thus, coming to the first remark, while (for our purposes) there is just one physical time value t , the reasoning to follow, about social space-time, emulates the above mentioned (special) relativity physics: For each particular event, there are many (fictitious) social time coordinate values, which one can label $\bar{x}^0$, $x^0(1)$, $x^0(2)$, $x^0(3)$, ..., that can be associated with the event. Each distinct value corresponds to a class of reference frames that are in motion relative to one or more other reference frames.

Moreover, each of those associated time coordinates has its *own* complete collection of space coordinates. In fact each such time coordinate has infinitely many such associated space coordinates because, for a given time axis, any particular set of space axes can be rotated through some arbitrary set of angles to produce a new set of space axes. Thus the complete specification of a reference frame requires selecting (a) origin, (b) measurement unit, (c) time axis, and (d) a particular set of space axes. In what follows in the present development, attention will initially be put on requirements (a), (b) and (c), with satisfaction of requirement (d) partly met, the remainder merely assumed, as when reference is made to “the” space coordinates. Eventually, however, while not accomplished here, (d) must also be fully met. As developed later, each moment of the history of each party in the social continuum is associated with a class of such reference frames, all of them having a common social time axis and differing only in their space axes. This common time axis will in fact correspond to ordinary physical time.

Now to the second remark, in what follows we introduce the convention that a lower case Greek letter may be taken as an index running over both time and space coordinates, for instance $\alpha = 0,1,...,N$; whereas, by contrast, in the same context lower case Latin letters will run over just the space coordinates, for instance $k = 1,2,...,N$. We also leave open the possibility that one of the two coordinate sets about to be named might (but need not) have the time coordinate previously labeled $\bar{x}^0$ in the above. Given two events in the social continuum, call them $P(1)$ and $P(2)$, suppose they have coordinates $x^\alpha(1) \equiv x^0(1), x^k(1)$ and $x^\alpha(2) \equiv x^0(2), x^k(2)$, respectively, in a reference frame S ; and coordinates $x'^\alpha(1) \equiv x'^0(1), x'^k(1)$ and

$x'^{\alpha}(2) \equiv x'^{0}(2), x'^{k}(2)$, respectively, in another reference frame S'. Now consider their differences

$$\Delta x^{\alpha} = x^{\alpha}(1) - x^{\alpha}(2) \text{ and}$$
$$\Delta x'^{\alpha} = x'^{\alpha}(1) - x'^{\alpha}(2) \tag{8.2}$$

as they appear in S and S', respectively . The second consideration about physical versus social time is: we assume that each set of (time or space) differences is connected to the other by respective linear transformations $T^{\alpha}{}_{\beta} : S \rightarrow S'$ and $T'^{\alpha}{}_{\beta} : S' \rightarrow S$ given by

$$\Delta x'^{\alpha} = \sum_{\beta} T^{\alpha}{}_{\beta} \Delta x^{\beta} \quad \text{, where } \alpha, \beta = 0,1,..., N \qquad (8.3),$$

and likewise with the roles of S and S' interchanged for $T'^{\alpha}{}_{\beta}$. In what follows, we assume that the transformation coefficients $T^{\alpha}{}_{\beta}$ and $T'^{\alpha}{}_{\beta}$ are dimensionless.

A third consideration about physical versus social time is we will assume that physical time t actually *is proportional to one of the social time coordinate values*—let it be designated by the one named above as $\bar{x}^{0}$ —and that the constant of proportionality is the fundamental signal speed, so that

$$ct = \bar{x}^{0} \text{ and } c\Delta t = \Delta \bar{x}^{0} \; . \tag{8.4}$$

That assumption will help provide, below, a connection between the abstract social space-time and the ordinary, physically real time as measured by physical clocks, in which social events occur.

<u>9. Measurement Units and Connection To Richardson Processes.</u>

For much of the discussion we will assume measurement using the time coordinate designated above as $\bar{x}^{0}$, equation (8.4), so the time coordinate must be units of time multiplied by the units of c; however, as noted in Part 8, we are treating the signal speed as a dimensionless number (a departure from physics), thus $\bar{x}^{0}$ will be in units of physical time as measured by ordinary clocks (times c which, as noted, we take to equal unity).

As previously mentioned, a complete reference frame will further require specifying one complete set of space coordinates to go with $\bar{x}^{0}$. For clarity any such reference frame will be called a <u>clock</u> reference frame and be designated by $\bar{S}$ which, thus, will actually stand for a class of such frames. Occasionally the discussion will invoke a more general frame S having unspecified social time x^{0}, as depicted in Figure 8.1; and we may iterate between $\bar{S}$ and S.

Considering such a more general S, in the discussion just preceding equation (8.1) reference was made to the 45-degree signal lines. Each point on such a line represents an <u>event</u>: namely the event of the signal passing through a point having a particular location at a particular time, as represented by coordinates x^α of that point; so a change in those coordinates in moving to a nearby event *on the signal line* can be represented as differences Δx^0_{sig} and Δx^i_{sig}, $i \geq 1$, in accordance with equations (8.2). If X^1 denotes the spatial axis, parallel to which the space-coordinates projection of a societal signal is moving, we can then write

$$\Delta x^1_{sig} / \Delta x^0_{sig} = 1 \ ,$$

in consequence of the 45-degree signal lines, or

$$\Delta x^1_{sig} = \Delta x^0_{sig} = ct \ \text{ and}$$
$$\Delta x^k_{sig} = 0 \,, \ k > 1 \ .$$

We assume that if the signal direction is not parallel to a spatial axis then one can always (by spatial rotation) pick new axes such that one of them is parallel to it.

However, we need to introduce a slightly more general assumption. Consider the sub-continuum formed just by the spatial coordinates x^k . It is such a sub-continuum if, for all pairs of events contained in it, all time differences between them vanish: $\Delta x^0 = 0$. In this sub-continuum any distances will be purely space distances. The more general assumption is that such purely space distances are given by the quantity

$$\Delta s = [\sum_k (\Delta x^k)^2]^{1/2} \ \text{, where } \ \Delta x^0 = 0 \ ;$$

that is, the sub-continuum is assumed to be Euclidean and the spatial axes are assumed to be mutually perpendicular. The preceding can be written in the equivalent form

$$\Delta s^2 = \sum_k (\Delta x^k)^2 \ . \tag{9.1}$$

The meaning of a distance in a space (sub-continuum), it is important to note, is such that, for a given set of coordinate differences, the quantity on the left-hand side of this equation is the same in *any* reference frame in which the space axes may have been rotated but for which the time axis is unchanged; that is, Δs , remains the same in all frames $\bar{S}$. (In Part 11 we will come to a generalization of this point in the discussion of an “interval in space-time”.)

The space projection of the movement of a societal signal is plainly such a purely space distance; thus the preceding equalities can be written more generally as

$$[\sum_k (\Delta x^k_{sig})^2]^{1/2} = \Delta x^0_{sig} \ , \tag{9.2}$$

true regardless of the direction of the signal, but from which the previous equations can be recovered (as the special case $\Delta x^k_{sig} = 0$, $k > 1$). In any such equation the units on either side must be equivalent. Equation (9.2) thus shows that one can use the unit of time differences as the unit of the space differences as well. Further, for the same reason and on account of the dimensionless linear transformation assumption (8.3) connecting any two reference frames, it must also be true that, in any other frame, the signal coordinate differences can be measured by the same time unit. Thus, from equations (8.2), in any reference frame the social signal coordinates themselves also can be so measured. In a clock reference frame, we have the added condition $x^0 = \bar{x}^0 \equiv ct$, for which the unit of measure is of physical time. Thus the unit of measure in equation (9.2) must be a physical time unit (or equivalent unit) in all reference frames. Finally, because we imagine the events to exist independently (alternatively, because a signal could have passed through *any* event in the continuum), one is free to use the same unit if the x^α describe some space-time event other than a signal. This brings one to conclude that any physical time unit may serve as a *unit of measure for social-spatial distances* in any reference frame. For example, if the unit for x^0 is [years], then the unit for the x^k is also [years]. (This is like the physics practice of measuring astronomical distances in light years. Note, while the units are the same in any reference frame, of course the numeric values need not be.) In what follows, except where otherwise stated, let us take the year as the unit of social space measure.

To recall the previous discussion of Richardson processes, in Part 7 we encountered the idea of a time delay in the recognition, by party 1, of a specific provocation by party 2. This led to the generalization of the party i partial reaction function to party j given by equation (7.9), in which $\varepsilon_{ij}(t)$ is the time delay in question. Reverting to the simplified notation of two parties 1 and 2, we now assume that $\varepsilon_{1,2}(t)$ is the *time required for signal transmission in the space-time continuum, from party 2 to party 1*, at the moment when party 1 is at time t as seen in any of the class $\bar{S}$ of clock frames. Figure 9.1 depicts such a frame where, again, just one spatial dimension is depicted. For simplicity, the origin is chosen to coincide with the location of party 1 at time $\bar{x}^0 = ct = 0$; thus, the event of reception is the place where the $\bar{X}^1$ and $\bar{X}^0$ axes intersect.

In what follows, I will refer to any transmitting party, such as party 2, as the sender and to any receiving party, such as party 1, as the receiver. (Note, especially in Part 15, the generalized use of these terms.) So we are saying that the sender (party 2), at the earlier time $t - \varepsilon_{1,2}(t)$ [$= -\varepsilon_{1,2}(0)$ in Figure 9.1] emitted the provocative signal received by party 1 at time t ($= 0$ in Figure 9.1 since, at moment of receipt, party 1 is located at the origin). In what follows this earlier time may also be termed the retarded time; and any quantity that originated at the earlier time relative to the receiver time, may be called a retarded quantity.

Similarly for all other parties acting as senders, we adopt the viewpoint that the delay in recognition of a provocation corresponds to signal transmission time in the space-time continuum:

$$\Delta x^0_{sig;1,2} = c\varepsilon_{1,2}(t) \tag{9.3}$$

which, with equation (9.2) shows the space distance traversed by the signal is given by

$$[\sum_k (\Delta x^k_{sig})^2]^{1/2} = c\varepsilon_{1,2}(t) = \varepsilon_{1,2}(t) \ . \tag{9.4}$$

That is (see the above discussion of units), the delay in recognition *also* equals the spatial distance between party 1 “now” (at time t) and where party 2 was “earlier”, the moment it emitted the provocation in question. The above allows one to partly characterize the location of the source (party 2, the sending party) in the space-time (namely at the moment when it emitted the provocation). (As previously mentioned, for N -many spatial dimensions, $N > 1$, empirically specifying the space coordinates remains to be done.)

The above corresponds in Figure 9.1 to various other parties as revealed by the transmission events located at the places and times at which the parties emitted the signals (provocations), all of which were received by party 1 where and when it was located at the origin (the place $x = 0$ and time $\bar{x}^0 = ct = 0$). (In addition, some parties are also represented by parts of their trajectories in the vicinity of their transmission events.) Accordingly, the indicated events are all located on one or the other of the two straight lines indicating signal transmissions converging from below at the origin, then diverging above it. We can generalize these two converging signal transmission lines as follows: In 2 space dimensions, the two lines would become a cone, with vertex at the origin; in 3 space dimensions, the cone would become a “hyper-cone”, which is difficult (or impossible) to visualize but can be represented mathematically. We refer generically to this hyper-cone, its specialization to fewer dimensions, and its divergent continuation above the origin, as the signal cone.

In sum, the figure depicts the simultaneous receipt by, and accumulative effect on, party 1, of the various provocations emitted at various previous times. This meshes with the Richardson process composition assumption first expressed in equation (1.1) and expressed, logarithmically, in equation (5.7); however, now the separate contributions from other parties $j \neq 1$, to the total reaction at time t of party 1, are each characterized by their corresponding delay times, $\varepsilon_{1,j}(t)$.

10. Further Discussion of the Constancy of Signal Transmission.

In Part 8 we introduced the assumption that the rate of signal transmission c is a constant which, furthermore, is the same, as viewed in every reference frame. In the preceding we have seen how the first part of that assumption leads to the signal cone construction of the various events received at a given time and place. In its second part (“... same ... in every ... frame”), the assumption also leads to a statement of what the social space-time idea logically requires, that special relativity theory provides. We will come to that in a moment.

Consider, first, the topological character of the motions of the parties. For a particular party, the collection of all the events representing it in the space-time diagram could form a partly or wholly disconnected set; or it could form a connected set; that is, a trajectory (solid line). If the latter, using t as a parameter, the trajectories would each be describing functions from the domain of all t -values to each component of the x -values, which we now assume they do. These functions might be differentiable (smooth), which here we also assume; by which, the functions are continuous. Let us consider a particular smooth trajectory, call it T_i .

Because T_i is smooth, a tangent line L is defined at each point of T_i . In what follows let us assume that, for each such actual or possible trajectory, for each such point P on it, the corresponding tangent line L functions as one of the time axes X^0 introduced in Part 8, and that P is the origin of this axis. (We include the possibility that two or more distinct points on the same or different trajectories might define the same L .) This implies that, at the moment corresponding to the point P , the party (actual or possible) having the trajectory T_i is co-moving with any reference frame containing this particular social time axis $X^0 = L$ (since, by the tangency condition, for an arbitrarily small region around P , the points comprising T_i are arbitrarily close to L).

[Though we do not need to be so formal in what follows, one could designate such time-axis-origin combinations $X^0(i,t_0)$ for the time axis and $P(i,t_0)$ for its origin, where i designates the party, the trajectory of which is generating the combination, and t_0 is the (physical) time of occurrence. Once complimentary space axes, call them $X^k(i,t_0)$, have also been chosen, then a complete reference frame thereby would be constructed, in which the indicated party i is at rest and located at the origin at the moment t_0 .]

We the viewers of Figure 9.1 implicitly are in a reference frame, one of the class $\overline{S}$, the time axis of which would be generated as the tangent line to such a trajectory with point of tangency at the origin. We take this viewpoint even if no actual trajectory corresponds to the origin of the moment; the possibility of such a trajectory requires that the type-$\overline{S}$ frame be considered. That we are momentarily at rest in this frame corresponds to the fact that the time axis $\overline{X}^0$ is vertical in the figure.

Now consider Figure 10.1, which is like the previous figure but with many details removed. In addition, trajectories T_1 and T_2 are drawn as curving or straight lines, respectively (within the limitations of my draw program; in general, all such trajectories may be curvilinear). As depicted, in a type-$\overline{S}$ reference frame the trajectory labeled T_1 generates various time axes at points along its extension, including the one shown in the figure and labeled $\overline{X}^0$, the tangent line with point of tangency at the origin O for the referent moment corresponding to O . (Again, even if no actual trajectory like T_1 generated this time axis, we carry out the discussion in order to accommodate that possibility.)

Given a signal such as the one depicted, starting at the point A , according to pre-relativistic (Euclidean / Newtonian) conventions, its speed is the ratio of physical time displacement to space displacement line segment lengths

$$c = [AB_1]/[B_1O] \ , \tag{10.1}$$

for segments defined by the endpoints B_1 , O , and A , respectively, and where c is the same quantity introduced in the discussion leading to equation (8.1). [Also, in equation (10.1) the numerator and denominator correspond, respectively, to the quantities described in equations (9.4) and (9.3).] In fact, already we have *defined* $[AB_1]$, the social space displacement between parties, as $[B_1O]$ (the signal delay between them) multiplied by c; for type $\overline{S}$ — the clock reference frames —we are merely repeating here the convention that social space displacement is ordinary physical time displacement (times c) in such frames. That is, for type $\overline{S}$ frames, we *keep* the pre-relativistic convention concerning the ratio of physical time displacement to space displacement.

Now consider the other trajectory, labeled T_2 . It generates the time axis corresponding to a (non-clock) reference frame in which receiver 2 is at rest, also with origin at O . By the conventional reckoning, which is wrong in this context, the space displacement from the moment of emission, event A , in this second frame should be the line segment having length $[AB_2]$; this quantity would replace $[AB_1]$ in (10.1), in order to get the corresponding expression relative to the rest frame of receiver 2 (again, wrong in this context). Putting this point another way, the space displacement should look different if the receiver is in motion (relative to our clock frame-type $\bar{S}$) because the party generating T_2 has changed position during the time required for the signal to bridge the gap between the points A and O . As drawn, *to party 2* the signal should appear to be moving more slowly (corresponding to $[AB_2] < [AB_1]$).

But, in the discussion leading to equation (8.1), we stipulated that the rate of signal transmission c is the same as viewed in every reference frame. Plainly, this stipulation cannot be sustained unless the time displacement, also, *relative to receiver 2* is allowed to be different from $[B_1O]$. That is, $c = [AB_2]/[B_1O]$ (***wrong***) *cannot* hold as a description of the situation from the viewpoint of party 2, because we have stipulated that an equation like (10.1) *does* hold: $c = \sum_k (\Delta x'^k_{sig})^2 / \Delta x'^0_{sig}$ (***correct***), by assumption; where numerator and denominator are the space and time displacements in the reference frame of receiver 2.

[Note that numerator and denominator correspond to, but differ in value from, the segments $[AB_1]$ and $[B_1O]$, respectively, of the clock frame represented in equation (10.1). Numerator and denominator in the correct equation are the analogues of the space and time quantities appearing in equations (9.4) and (9.3), respectively, applying in a clock frame.]

The same reasoning applies to the reference frame of any other party in motion, relative to the clock frames $\bar{S}$. Thus the conventional definition of time displacements (and, as a consequence it turns out, space displacements also—see the prime mark in the numerator, above) must be altered, but in a way that retains the conventional value, relative to the clock frames $\bar{S}$ in which we, the viewers, are regarded to be motionless. The equivalent to the problem, how to accomplish this, was encountered in physics and solved by special relativity. (In the physical exemplar the problem included anomalous empirical observations, of which the experiments by Michelson and Morley are the most widely known; in the present societal analogue the problem is only conceptual.) This solution invokes the concept of a space-time metric, to the societal-model transcription of which we now turn.

11. Metric of Social Space-Time.

As previously noted, for a sub-continuum, between all points of which the coordinate differences are constrained by $\Delta x^0 = 0$, the distances are assumed to be given by equation (9.1), above. What about distances in the more general case of unconstrained Δx^0 's? The distance-defining equation of a space is conventionally called the metric of the space. So the question is asking for the metric of the social space-time.

One can start by assuming a particular metric; alternatively, the form of the metric can be regarded to follow as a conclusion from *apriori* assumptions. A statement of suitable prior assumptions (Rohrlich, 1990, p. 267) is instructive:

> Let x , y , z and x' , y' , z' be the rectilinear orthogonal Cartesian coordinates of two reference systems S and S' . Let t and t' be their respective time coordinates.

> Assume that S and S' are in relative motion with constant velocity **v** . We are seeking the transformation from S to S' subject to the following restrictions.
>
> (1) If a particle is in uniform motion relative to S , it must be in uniform motion also relative to S' for all values of the coordinates. (Equivalence of inertial systems in uniform relative motion.)
>
> (2) A [signal] wave in a vacuum expanding spherically with velocity c relative to S must expand with velocity c also relative to S' . (Independence of [signal] velocity from source and observer motion.)

In the above quotation, to fit the present application I have used the term “signal” in place of the word “light” that was used in the original. The term source has the same usage as “sender”. Following the opening paragraph of Part 8, in the above we also identify a “particle” with a (societal) party; provisionally a nation state. In the above quotation, item (1) corresponds to our equation (8.3); item (2) corresponds to our extensive discussion in the above, concerning the constancy of c .

The above considerations lead [Rohrlich, 1990, p.271, equation (A1-18)] to the metric given below. (While Rohrlich’s notation differs from below—his shows coordinates x , y , z , and t rather than coordinate differences—the meaning is the same because his coordinates are measured relative to a point at the origin x', y', z', $t' = 0$.) In our notation, that metric is

$$\Delta s^2 = \sum_k (\Delta x^k)^2 - (\Delta x^0)^2 , \tag{11.1}$$

for coordinate differences Δx^α , where the coordinate differences are defined the same way as in equations (8.2). This equation is the (physical) space-time metric of special relativity, where it is known as the Minkowski metric. I take it for use here. A distance between a pair of events defined by this metric is called an interval, by which I will refer, below, to space-time distances. In Part 9 we saw that the meaning of the distance in a space sub-continuum is that the quantity on the left in equation (9.1) remains unchanged with changes in the purely space axes. The generalization, here, is that permissible changes, in both space and time axes, from one reference frame to another must be such that the interval, given by (11.1), between two events in space-time remains unchanged. The underlying idea is the same as for ordinary distance: that which is objective, namely (in this context) the interval between two events, should have the same value regardless of the coordinates to which the events are referred.

The transition from a Euclidean distance to a Minkowski interval also introduces the novelty that, evidently, $\Delta s^2 < 0$ is permitted by equation (11.1), as happens whenever the squared time difference term is greater than the sum of the squared space difference terms. In that case the interval must be the square root of a negative quantity, thus the interval must be an imaginary number. Wherever appropriate below I will follow the naming convention (in physics) and refer to any imaginary interval as timelike; and I will refer to any non-zero real valued interval, for which $\Delta s^2 > 0$, as spacelike, and to any interval for which $\Delta s^2 = 0$, as null. Finally, I follow another convention and use the term displacements to refer to coordinate difference terms such as those in equation (11.1).

[Note that many discussions in physics literature concerning the above reverse the signs on the right hand side of equation (11.1) so that the timelike intervals are real valued and the spacelike, imaginary. The two alternative conventions, imaginary versus real timelike values, give entirely equivalent results when one or the other is followed consistently.]

It is useful to express equation (11.1) in different notation. Let $\eta_{\mu\upsilon}$ be an array of numbers given by

$$\eta_{\mu\upsilon} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11.2}$$

where the Greek subscripts follow the domain convention described in the paragraph preceding equation (8.2), above. The first named subscript denotes the matrix row; the second subscript, the matrix column, so that, for instance, $\eta_{00} = -1$ and $\eta_{01} = 0$. In the above, the quantities symbolized by $\eta_{\mu\upsilon}$ are together called the metric tensor, by which I will refer to them below. With the aid of (11.2), equation (11.1) can be written

$$\Delta s^2 = \sum_{\mu,\upsilon} \eta_{\mu\upsilon} \Delta x^{\mu} \Delta x^{\upsilon} \ ,$$

which conventionally is written with the summation sign omitted (summation convention), as

$$\Delta s^2 = \eta_{\mu\upsilon} \Delta x^{\mu} \Delta x^{\upsilon} \ . \tag{11.3}$$

The tip-off that summation is to be performed is the double occurrence of each of the two indices, once in the displacements Δx^{α} and once in the metric tensor. Also useful is a third form of the space-time metric. Let a new form of the displacements be defined by

$$\Delta x_{\upsilon} \equiv \eta_{\mu\upsilon} \Delta x^{\mu} \equiv \sum_{\mu} \eta_{\mu\upsilon} \Delta x^{\mu} \ , \tag{11.4}$$

where, again, the double occurrence of an index, one of them on the metric tensor, signifies summation. Using equations (11.3) and (11.4), the third metric formula is

$$\Delta s^2 = \Delta x_{\upsilon} \Delta x^{\upsilon} \equiv \sum_{\upsilon} \Delta x_{\upsilon} \Delta x^{\upsilon} \ . \tag{11.5}$$

Here, the metric tensor does not explicitly appear, yet the appearance of the index as both subscript and superscript signifies that one of them has been obtained from the other in the manner of equation (11.4); this implicit use of the metric tensor again signifies that summation is to be performed on the doubly occurring index. Also useful are the differential forms of any of equations (11.1), (11.3), (11.4) and (11.5); for example the last one is written

$$ds^2 = dx_\upsilon dx^\upsilon .$$

(11.6)

Two generalizations are needed. The first is to define the concept of a space-time vector. Using the assumptions given in the long quotation appearing above, one can infer (Rohrlich, 1990, p. 267ff) the specific values required of the linear transformations $T^\alpha{}_\beta : S \to S'$ and $T'^\alpha{}_\beta : S' \to S$ named in equation (8.3). In accordance with the discussion following equation (11.1), a criterion for such transformations is that they preserve the interval between any two events. In general such transformations can be regarded as the product of two partial transformations. The novel part addresses the space and time issues mentioned above; it is called a (special) Lorentz transformation. The other contributing part to the transformation addresses ordinary translations and rotations in the Euclidean sub-continuum. We will refer to the general transformation (the product, i.e. the combined application, of the two partial transformations) as the generalized Lorentz transformation or simply, where no confusion could arise, as the Lorentz transformation.

If any other quantity whatsoever has components A^α all of which transform between two reference frames, say S and S' exactly like the displacements (i.e. by Lorentz transformations); that is, if

$$A'^\alpha = \sum_\beta T^\alpha{}_\beta A^\beta \equiv T^\alpha{}_\beta A^\beta \ ,$$

(11.7)

then A^α is called a (space-time) vector, where the indicated summation convention involving the transformation emulates the one involving the metric tensor. Evidently, displacement is (by definition) such a vector.

The second generalization concerns the metric, with particular reference to the form expressed by equations (11.5) and (11.6). Let A^α and B^α be any two vectors. Let their inner product be defined by

$$(A,B) \equiv A_\upsilon B^\upsilon = A_0 B^0 + A_k B^k$$

(11.8)

For the summation convention involving the Greek indices, refer to the above. The final right hand term uses the same idea except summation is over the space indices $k = 1,2,..., N$, only, the time component being written separately. This separation is a convenient way to distinguish time from space components; and the final term (or understood summation of terms) $A_k B^k$ may be written by itself. The above conventions will be used henceforth.

It can be shown that inner products are preserved in Lorentz transformations [again, those that preserve the intervals given by equation (11.5) or its equivalent]:

$$A'_\upsilon B'^\upsilon = A_\upsilon B^\upsilon ,$$

(11.9)

where primed and unprimed quantities name the components of the corresponding vectors in any two reference frames S' and S , respectively, connected by Lorentz transformations.

An important specialization of inner product is the vector magnitude $|A|$, given (in squared form) by

$$|A|^2 \equiv A_\upsilon A^\upsilon ,$$
(11.10)

the inner product of the vector A with itself, the magnitude thus being the positive square root of the quantity on the left. Note that the meaning of (11.10) is given by the metric equation (11.1) generalized, here, to the components of any vector having squared space and time parts $\sum_k (A^k)^2$ and $(A^0)^2$, respectively.

[Thus A^α may, or may not, be a displacement vector. When it is, that is when $A^\alpha = \Delta x^\alpha$, equation (11.10) specializes back to equation (11.5).]

Just as in the discussion following equation (11.1), the squared magnitude on the left of (11.10) can be negative, positive, or zero; again leading to square roots that are imaginary, non-zero real valued, and vanishing, respectively; with corresponding vectors denoted by the labels timelike, spacelike, and null.

A further point concerns the time axis x^0 of the corresponding reference frame S . Let the subscript $_{//}$ denote displacements parallel to X^0 . Such displacements are at rest in S , thus they are characterized by $\Delta x^i{}_{//} = 0$ and $dx^i{}_{//} = 0$ in equations (11.5) and (11.6), respectively. Thus (11.6), for example, becomes

$$ds^2 = dx_{0,//} dx^0{}_{//} .$$

From equations (11.2) and (11.4), one finds that $dx_0 = -dx^0$ (for any displacement), which yields

$$ds = i dx^0{}_{//} \text{ and space displacements vanish: } dx^i{}_{//} = 0 \text{ , and}$$
$$ds = i d\bar{x}^0{}_{//} = icdt \text{ and space displacements vanish,}$$
(11.11)

where the second line uses equation (8.4) and applies to $\bar{S}$ -type frames, only.

Returning to the general case of any reference frame, a further assumption

$$\Delta s^2, ds^2 < 0 \text{ , } s \text{ defined along the path of a party,}$$
(11.12)

constrains the values of intervals that are measuring the trajectories of the parties; from (11.12) evidently they are to be imaginary numbers, which is consistent with equation (11.11). That is to say, *trajectories of social parties are assumed to be timelike*. [See the discussion following equation (11.1).]

At this point let us digress a moment to note an important implication of equation (11.12) for the structure of the Richardson process. Part 9 introduced the assumption that the time delay $\varepsilon_{1,2}(t)$ in the generalized Richardson process, between the provocation by a party 2 and its recognition by a party 1, equals the time required for signal transmission in the space-time continuum, from party 2 to party 1, such that the signal arrives at the moment when party 1 is at time t as seen in any clock frame $\bar{S}$. Consider the metric equation (11.1) referred to $\bar{S}$. (Recall that frames $\bar{S}$ differ only in their space axes; so t is the same in all of them—a very good thing, since the latter is referring to physical time, as previously discussed.)

With the aid of equation (8.4), that metric can be written

$$\Delta s^2 = \sum_k (\Delta \bar{x}^k)^2 - (c\Delta t)^2 \ .$$

(11.1)'

Comparison with (11.12) shows $\sum_k (\Delta \bar{x}^k)^2 - (c\Delta t)^2 < 0$ from which, in view of equation (8.1),

$$\sum_k (\Delta \bar{x}^k)^2 < (c\Delta t)^2 = \Delta t^2 \ .$$

(11.13)

Now suppose the left-hand side of equation (11.13) refers to the distance traveled by the provocative party—which for the purpose of this argument we will assume to be moving toward the responding party—during the time required for the provocation signal to reach the responding party. Then, by the time-delay assumption in Part 9, $\Delta t = \varepsilon_{1,2}(t)$ and (11.13) becomes

$$\sum_k (\Delta \bar{x}^k)^2 < [\varepsilon_{1,2}(t)]^2 \ .$$

(11.14)

In the case of any motion (not just toward the responder), this inequality is preserved because the component of spacelike displacement toward the responder must be even smaller than the total spacelike displacement. As just mentioned, $\varepsilon_{1,2}(t)$ is the time of transmission from a sender to a receiver, relative to a frame $\bar{S}$, but, from the discussion in Part 9, particularly equation (9.2), this transmission time is also the spatial distance as seen in $\bar{S}$ from the position of sender at the moment T of transmission (provocation) to the spatial position of receiver at the moment R of reception (response). Since the left-hand side of expression (11.14) is the corresponding amount by which the spacelike distance of sender to receiver has changed from moment T to moment R, this position change is also the amount by which the transmission delay has *changed* from T to R; that is $[\sum_k (\Delta \bar{x}^k)^2]^{1/2} = \Delta\varepsilon_{1,2}$. Therefore (11.14) becomes

$$\Delta\varepsilon(t)_{1,2} < \varepsilon_{1,2}(t) \ .$$

(11.15)

Since (in the limit of infinitesimal displacements) $\dot{\varepsilon}(t)_{1,2} = \Delta\varepsilon(t)_{1,2} / \varepsilon_{1,2}(t)$, equation (11.15) shows

$$\dot{\varepsilon} < 1 \ . \tag{11.16}$$

(Note this is the result used in Part 7.) Compare this result with the expression $[1-\dot{\varepsilon}_{1,2}(t)]$ appearing in equation (7.7). Looking at the latter equation might suggest that, for sufficiently great a rate of change in the delay term, the sign of the fatigue and reaction coefficients might be reversed, converting positive reaction and negative fatigue coefficients to negative and positive, respectively. Equation (11.16) shows that this cannot happen. As the above has shown, essentially this results from the confinement of parties in the social space to speeds less than the fundamental signal speed c .

Returning to the main line of discussion, from the first paragraph of Part 9, we know that the unit of $d\bar{x}^0$ is clock time; thus the unit of ds also is clock time.

Finally, let us introduce the following convention; unless otherwise stated: unprimed vector components will refer to the unprimed general frame S ; primed components will refer to another general frame S' ; components with the over-score, for instance $\bar{x}$, will refer to the clock frames $\bar{S}$.

12. Space-Time Formulation of Richardson Processes.

Now we are going to write an equation using an inner product involving certain vectors. We then will equate the various parts of the generalized Richardson process equation (7.10) to parts of this equation. The result will be that the generalized Richardson process is seen as equivalent to a special case of the space-time equation involving the inner product. Thus the latter will be the space-time representation of Richardson processes.

The details are messy but the underlying idea is simple: the magnitude of provocation emitted by a given party in the Richardson process is to be identified with the space part of the magnitude of velocity of that same party, as seen in the clock frames $\bar{S}$; and the rate of change in the provocation is to be identified with the space part of the acceleration of that party, again as seen in $\bar{S}$.

The first step is to identify the vectors. We define the parameter

$$s = \int_0^a ds \ , \ a \text{ marking the “present” moment,} \tag{12.1}$$

of which the coordinates are regarded as functions, $x^\alpha(s)$. Now consider the path (trajectory) of a given party. On such a path, s functions as a parametric representation of the path; namely, s gives the path length. (It is as if one laid down a tape measure along the path, with s proportional to the reading in number of units on the tape.) Using equations (11.1)' and (11.12), we conclude $i \cdot s$ is a strictly increasing monotonic function of t :

$$\frac{i \cdot ds}{dt} > 0 .$$

(12.2)

[This result resembles but is distinct from the one concerning the strict monotonicity of u and t , following equation (7.4).]

Next, define velocity, acceleration, and spacelike force by

$$v^{\alpha} \equiv dx^{\alpha} / ds \equiv \hat{x}^{\alpha} ,$$
$$a^{\alpha} \equiv dv^{\alpha} / ds \equiv \hat{v}^{\alpha} ,$$
$$f^{\alpha}(k) \equiv m(k) \cdot a^{\alpha}(k) , \; m(k) > 0 ,$$

(12.3)

respectively, where $m(k)$ denotes a constant specific to the k[th] party; it will be the analogue to the mass of a particle. If, for a given reference frame S , the velocity of a given party is constrained by $v^{k} = 0$, then S will be called a rest frame of the party.

[Concerning $f^{\alpha}(k)$, there is an important qualification to be stated, which I insert as the final paragraph of part 15, below. Concerning the expression "spacelike force," from the Newtonian conception of space and time we are accustomed to thinking of the force defined above as the only type. In the above, the term "spacelike" qualifies that type because of a different type, the "radiation reaction" force, mentioned only in passing following equation (17.8), below, but key to any dynamic simulation that might be undertaken. As concluded in equation (12.6), below, the spacelike force has vanishing time component in the rest frame of the party to which the force is applied; thus the name—my own terminology; apparently it has no special name in physics literature. It should be noted that, in physical discussions, the customary manner of treating the third of equations (12.3) would to define the left-hand side with a logically independent property, e.g. using Newton's gravitation law or Einstein's generalization of it, or Hooke's law for springs, or the Lorentz electromagnetic force law, etc., so that the equation is empirically falsifiable. However, in the above the two sides are equal by definition. Thus, the falsifiable aspect requires that the left-hand side further be identified with some other empirically independent quantity; and this remains to be done.]

Suppose that the space differential displacements vanish: $dx^{i} = 0$. Then, from the first of equations (11.11) and (12.1), we have

$$s = ix^{0} + \text{constant} .$$

The first of the following pair of equations follows from equation (8.4), alternately from the second of (11.11), and applies only in the clock frames. The second of the pair follows from the assumption (11.12). We find

$$s = i\bar{x}^{0} + \text{constant} = ict + \text{constant} \text{ (\textit{type } } \bar{S} \text{ \textit{frames only}) and}$$
$$s^{2} < 0 \text{ (any frame } S \text{)}.$$

(12.4)

The first of these involves clock time, thus it applies only in the $\bar{S}$ class of frames. However, in view of the transformation (11.7) between any two reference frames, the measurement unit of s [years] must be the same in any frame. The second of (12.4) shows that the value of s is imaginary *along any actual path* taken by the referent party in the continuum. Because s is the same in every reference frame S (see the discussion following equation 11.1), these two conclusions about unit and path apply in every such frame.

For later reference we further note the following: From equation (11.6) one can deduce that (in any frame) the above social velocity and acceleration vectors are orthogonal, in reference to the same party at a given point on its trajectory,

$$a_\alpha v^\alpha = 0 \ , \tag{12.5}$$

which shows that, for a party relative to its own rest frame S': $v'^k = 0$, the time component of acceleration vanishes and, from the third of (12.3), likewise for the force applied to it,

$$a'^0 = 0 \, ,$$
$$f'^0 = 0 \ \text{(both equations in rest frame).} \tag{12.6}$$

Now we want to focus on the clock frames $\bar{S}$, with reference to a party k. (Note that, in this usage, this letter plays a similar role as previously played by the letter i, starting in Part 1; namely, both are labeling one of the parties in developing the relationships between pairs of parties.)

Through the remainder of Part 12, I omit the over-score marking from vector components in $\bar{S}$. We regard party k in its role as a receiver of signals (provocations) from other parties (i.e. from senders), the typical one of which we denote by j. When discussing all parties without distinguishing receiver from senders, I will use the index ℓ. Further, let N denote the number of sending parties, so that the total number of parties, including the receiving party, is $N+1$.

As noted above in Part 8, the space axes of $\bar{S}$ are not yet specified. We specify one of those axes now and label the result $\bar{S}_k$. Let $f(k,s)$ denote the force [defined in (12.3)] on k at the moment indexed by the parameter s; let $f_\perp(k,s)$ be the part of $f(k,s)$ perpendicular to $\bar{X}^0$; and let $f_i(k)$ be the components of $f_\perp(k,s)$. These components form the projection of the force vector onto the space sub-continuum. From this we can choose space axes such that one of them— let it be the axis labeled 1 —is *parallel to this projection*; call this axis $\bar{X}^1$. By this definition

$$f_i(k) = 0 \ , \ i > 1 \ . \tag{12.7}$$

Let $v(\ell,k,s'_\ell)_{proj}$ be a certain projection of the velocity $v(\ell,s'_\ell)$ of party ℓ which originated at the retarded time indexed by $s'_\ell \le s$; namely, this projection is onto the space sub-continuum of the clock frame $\bar{S}_k$. The condition $\le$ reflects that one of the projections onto $\bar{S}_k$ may come from k , itself, for which the time retardation would vanish. Let $v^i(\ell,k,s'_\ell)_{proj}$ be the space components in $\bar{S}_k$ of this projection. Next, define a new quantity

$$(f,v) \equiv f_i(k) \cdot v^i(\ell,k,s'_\ell)_{proj} \;, \quad s'_\ell \le s \;. \tag{12.8}$$

This is equivalent to

$$(f,v) = f_i(k) \cdot v^i(\ell,k,t'_\ell)_{proj} \;, \quad t'_\ell \le t \;, \tag{12.9}$$

because of the previous conclusion, equation (12.2), that s is an increasing monotonic function of t ; likewise for s'_ℓ versus t'_ℓ . (To anticipate, below I will propose to identify t' with $u = f(t)$ in the elaboration of Abelson's approach that we considered earlier.)

To aid readability in what immediately follows, I omit the time arguments. From (12.7), equation (12.9) becomes $(f,v) = f_1(k) \cdot v^1(\ell,k)_{proj}$. From equations (11.2) and (11.4), one finds that $f_1 = f^1$, thus equation (12.9) can be written as

$$(f,v) = f^1(k) \cdot v^1(\ell,k)_{proj} \;. \tag{12.9a}$$

As previously discussed (parts 8 and 9), the clock frames differ only in the choice of space axes; the time axis $\bar{X}^0$ is common to all of them; thus the magnitude $|v(\ell,k)|_{proj}$ of $v(\ell,k)_{proj}$ [i.e. of the space part of $v(\ell)$, see equation (9.1)] is independent of the choice of k , thus the index may be omitted:

$$|v(\ell,k)|_{proj} = |v(\ell)|_{proj} \;, \quad \ell = 1,\ldots,N,N+1 \;,$$

Focusing on values of $\ell \ne k$, i.e. on the senders j , in $\bar{S}_k$, $\cos\alpha_{jk} = v^1(j,k)_{proj} / |v(j)|_{proj}$ or

$$v^1(j,k)_{proj} = |v(j)|_{proj} \cdot \cos\alpha_{jk}$$

where α_{jk} is the angle between $v(j,k)_{proj}$ and the axis $\bar{X}^1$ of the clock frame $\bar{S}_k$. Combining all the above we find that equation (12.9a) can be written (for $\ell =$ any of the j)

$$(f,v) = f^1(k) \cdot |v(j)|_{proj} \cdot \cos\alpha_{jk}$$

which, using the second and third of equations (12.3), can be written

$$(f,v) = m(k)\cdot \hat{v}^1(k)\cdot |v(j)|_{proj}\cdot \cos\alpha_{jk}$$

from which

$$(f,v) = m(k)\cdot \dot{v}^1(k)\left[\frac{dt}{ds_k}\right]\cdot |v(j)|_{proj}\cdot \cos\alpha_{jk} \quad ,$$

where ds_k is the invariant interval [equations (11.6) and (12.1)] describing the path of receiver k. Now the discussion leading to the replacement of $v^1(j,k)$ by $|v(j)|_{proj}\cdot \cos\alpha_{jk}$, above, works also for the referent receiver k:

$$v^1(k) = |v(k)|_{proj}\cdot \cos\alpha_k$$

where $v^1(k)$ is the component of the velocity of receiver k along the axis $\overline{X}^1$ of the clock frame $\overline{S}_k$, $v(k)_{proj}$ is the spacelike projection in $\overline{S}_k$ of the velocity of k, and α_k is the angle between $v(k)_{proj}$ and $\overline{X}^1$. [This angle may be non-vanishing since $v(k)_{proj}$ need not be parallel with the spacelike component of the force applied to k.] Thus we can write

$$\dot{v}^1(k) = \frac{d}{dt}|v(k)|_{proj}\cdot \cos\alpha_k = \left[\frac{d}{dt}|v(k)|_{proj}\right]\cos\alpha_k - \dot{\alpha}_k |v(k)|_{proj}\sin\alpha_k \quad ,$$

by which the previous expressions for (f,v), starting with equation (12.9a), become

$$(f,v) = m(k)\cdot\left[\left[\frac{d}{dt}|v(k)|_{proj}\right]\cos\alpha_k - \dot{\alpha}_k |v(k)|_{proj}\sin\alpha_k\right]\cdot\left[\frac{dt}{ds_k}\right]\cdot |v(\ell)|_{proj}\cdot\cos\alpha_{jk} \quad ,$$

or

$$\left[\frac{(f,v)}{m(k)}\right]\cdot\left[\frac{ds_k}{dt}\right] + \dot{\alpha}_k |v(k)|_{proj}\cdot |v(\ell)|_{proj}(\sin\alpha_k)(\cos\alpha_{jk})$$

$$= \left[\frac{d}{dt}|v(k)|_{proj}\right]\cdot |v(\ell)|_{proj}(\cos a_k)(\cos\alpha_{jk}) \quad .$$

(12.10)

As before, except where otherwise stated, in the following $\ell = k$ is a permissible value of ℓ. (Note this creates redundant notation since α_k means the same thing as α_{kk}.) From equation (12.10) one finds

$$\left|v(\ell)\right|_{proj}\cdot\left[\frac{(f,v)}{m(k)(\cos a_k)(\cos\alpha_{\ell k})}\right]\cdot\left[\frac{ds_k}{dt}\right]+\dot{\alpha}_k(\tan\alpha_k)\left|v(k)\right|_{proj}\cdot\left|v(\ell)\right|^2{}_{proj}$$

$$=\left[\frac{d}{dt}\left|v(k)\right|_{proj}\right]\cdot\left|v(\ell)\right|^2{}_{proj}\text{ , where }\ell = j\text{ or }k\text{ ; }\alpha_k\text{ , }\alpha_{jk}\neq\pm\pi/2\text{ .}$$

Various restrictions apply. First is that $f_1(k)$ be non-vanishing. This restriction corresponds to $\bar{S}_k$ being well defined, because the axis $\bar{X}^1$ in $\bar{S}_k$ is to be chosen parallel to $f_1(k)$, which orientation is meaningful only if the vector component $f_1(k)$ is non-vanishing. In turn, $f_1(k)\neq 0$ is assured provided the magnitude of $f(k)$ (the force on k) is non-vanishing.

[This assurance comes about as follows. In the rest frame S' of k, the second of equations (12.6) applies. From equations (11.2) and (11.10), the discussion concerning (11.7), and the stipulation that $f(k)$ be a vector, it follows that $\left|f(k)\right|^2=\sum_{i=1}^{3}[f'^i(k)]^2\geq 0$. By prior choice of the axis $\bar{X}^1$, we have $\bar{f}^i=0$, $i>1$; thus the magnitude of the force vector in the frame $\bar{S}_k$ is given by $\left|f(k)\right|^2=(\bar{f}^1)^2-(\bar{f}^0)^2$. Combining the above two expressions shows $(\bar{f}^1)^2-(\bar{f}^0)^2\geq 0$ or $0\leq(\bar{f}^0)^2<(\bar{f}^1)^2$. The latter shows that if any part of the force is non-vanishing then $f_1(k)=f^1(k)$ — see discussion leading to (12.9a) —must also be non-vanishing. The short reason is, the force in question is a spacelike vector.]

Second, the above stated restrictions on α_k and the α_{jk} follow from the trigonometric functions appearing on the left-hand side. They reflect the inability of the spacelike projection in $\bar{S}_k$ of the receiver velocity to generate a non-zero component along any candidate $\bar{X}^1$ if the two are mutually perpendicular and, likewise, for the spacelike projections in $\bar{S}_k$ of sender velocities.

Next, provided the above restrictions are met, define

$$\xi(\ell,k)\equiv\left[\frac{(f,v)}{m(k)(\cos a_k)(\cos\alpha_{jk})}\right]\cdot\left[\frac{ds_k}{dt}\right].$$

Equation (12.10) then becomes equivalent to

$$\left|v(\ell)\right|_{proj}\cdot\xi(\ell,k)+\dot{\alpha}_k(\tan\alpha_k)\left|v(k)\right|_{proj}\cdot\left|v(\ell)\right|^2{}_{proj}=\left[\frac{d}{dt}\left|v(k)\right|_{proj}\right]\cdot\left|v(\ell)\right|^2{}_{proj}\text{ ,}$$

$$\ell = j\text{ or }k\text{ ; }\alpha_k\text{ , }\alpha_{jk}\neq\pm\pi/2\text{ .}\tag{12.11}$$

In what follows the above indicated restrictions on α_k and α_{jk} continue to apply except where otherwise stated. [Starting with equation (12.12) the second restriction will be removed.] Let N be the total number of sending parties in the system (excluding the receiving party). Now we can think of the index p as ranging across the receiver k plus exactly the M -many sender parties for which the restriction on α_{jk} holds, $M \le N$, to form a sequence of equations like (12.11) and, on those equations, we can sum across all such values p :

$$\sum_{p=1}^{M+1} |v(p)|_{proj} \cdot \xi(p,k) + \dot{\alpha}_k (\tan \alpha_k) |v(k)|_{proj} \cdot \sum_{p=1}^{M+1} |v(p)|^2{}_{proj}$$
$$= \left[\frac{d}{dt} |v(k)|_{proj} \right] \cdot \sum_{p=1}^{M+1} |v(p)|^2{}_{proj} \;\; ,$$

which, dividing both sides by $\sum_{p=1}^{M+1} |v(p)|^2{}_{proj}$, gives

$$[\sum_{p=1}^{M+1} |v(p)|_{proj} \cdot \xi(p,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj}] + \dot{\alpha}_k |v(k)|_{proj} \tan \alpha_k$$
$$= \frac{d}{dt} |v(k)|_{proj} \;\; .$$

This has the restriction $\alpha_{jk} \ne \pm\pi/2$. If we define $\xi(j,k) \equiv 0$ for the case $\alpha_{jk} = \pm\pi/2$, where $j \ne k$, then the above can also be written

$$\sum_{j=1}^{N} |v(j)|_{proj} \cdot \xi(j,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj}$$
$$+ |v(k)|_{proj} \cdot \xi(k,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj} + \dot{\alpha}_k |v(k)|_{proj} \tan \alpha_k$$
$$= \frac{d}{dt} |v(k)|_{proj} \text{, where } j \ne k \;,\; \alpha_k \ne \pm\pi/2 \;, \tag{12.12}$$

where the distinction between receiver k and senders j has been reintroduced. (That is, the term involving $\xi(k,k)$ on the left-hand side has been separated from the terms involving $\xi(j,k)$, $j \ne k$.) The following expressions retain the indicated restriction on α_k ; the restriction on the α_{kj} for $j \ne k$ is removed.

Now we are ready to connect the above to the previous development of the Richardson process. Equation (7.10) can also be expressed as

$$\dot{z}_k(t) = \sum_{j=1}^{N} [a'_{M,kj}(u_{k,j})] \cdot [z_j(u_{k,j})] - b_k [\sum_{j=1}^{N} \dot{u}_{kj}(t) \cdot z_{kj}(t)] + g_{M,k}(t) \;\;,\;\; j \ne k \;\;,$$

where k has replaced i as the receiver index. Using the definition of u_{kj} [$\leftrightarrow u_{ij} \equiv t - \varepsilon_{ij}$ in equation (7.9)] gives $\dot{u}_{kj} = 1 - \dot{\varepsilon}_{kj}$. With this substitution and using (5.8) (with k replacing i), the preceding becomes

$$\dot{z}_k = [\sum_{j=1}^{N} a'_{M,ij}(u_{k,j}) \cdot z_j(u_{k,j})] - b_k z_k + b_k (\sum_{j=1}^{N} \dot{\varepsilon}_{kj} \cdot z_{kj}) + g_{M,k} \ , \ j \neq k \ . \tag{12.13}$$

Recalling the discussion, next to last paragraph in Part 5, we are free to choose a value for $g_{M,k}$. With appropriate restriction we also are free to equate (in a manner consistent with the variable domains—non-negative real variables equated to the same, etc.) the provocation values $z_l(t)$ and the delayed provocation times u to the space-time quantities developed in this present section. Let the grievance values be given by

$$g_{M,k} = \pm[\dot{\alpha}_k |v(k)|_{proj} \tan\alpha_k] - b_k \sum_{l=1}^{N} \dot{\varepsilon}_{kl} z_{kl} \ , \tag{12.14}$$

$$\pm z_l = |v(l)|_{proj} \text{, for all parties } l = 1,2,\ldots,N \text{, and} \tag{12.15}$$

$$u_{kj} = t'_j \ . \tag{12.16}$$

In equation (12.15), if $z_l < 0$ the negative sign is to be chosen; otherwise choose the positive sign. In (12.14), choose the sign to agree with the sign with respect to receiver k in (12.15) for $\ell = k$. [Note that, in (12.14), the final term on the right serves to subtract the like term in (12.13).]

From (12.14)

$$\dot{\alpha}_k |v(k)|_{proj} \tan\alpha_k = \pm b_k [\sum_{l=1}^{N} \dot{\varepsilon}_{kl} z_{kl}] \pm g_{M,k} \ , \ k \neq j \ ; \tag{12.17}$$

and from (12.15)

$$\pm \dot{z}_k = \frac{d}{dt} |v(k)|_{proj} \ . \tag{12.18}$$

for receiver k , where the sign choice follows that of z_k . Using (12.15) through (12.18), equation (12.12) can be written

$$\sum_{j=1}^{N} z_j(u_{kj}) \cdot [\pm\xi(j,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj}]$$
$$+ z_k(t) \cdot [\pm\xi(k,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj}] \pm b_k [\sum_{l=1}^{N} \dot{\varepsilon}_{kl} z_{kl}] \pm g_{M,i}$$
$$= \pm \frac{d}{dt} z_k(t), \;\; j \neq k$$

(12.19)

Now consider two possible cases.

Case (a). $z_k \geq 0$: Equation (12.19) is

$$[\sum_{j=1}^{N} z_j(u_{kj}) \cdot [\pm\xi(j,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj}]$$
$$+ z_k(t) \cdot [\xi(k,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj}] + b_k [\sum_{l=1}^{N} \dot{\varepsilon}_{kl}(t) \cdot z_{kl}(t)] + g_{M,k}(t)$$
$$= \frac{d}{dt} z_k = \dot{z}_k, \;\; j \neq k \;\; .$$

(12.20a)

Comparison of the above with equation (12.13) shows the Richardson process reaction coefficients are given by

$$a'_{M,ij}(u_{k,j}) = \pm \xi(j,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj} \quad j \neq k \;\; \text{(reaction coefficients)}$$

(12.21a)

Case (b). $z_k < 0$: Equation (12.19) now becomes

$$[\sum_{j=1}^{N} z_j(u_{kj}) \cdot [\pm\xi(j,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj}]$$
$$- z_k \cdot [\xi(k,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj}] - b_k [\sum_{l=1}^{N} \dot{\varepsilon}_{kl} \cdot z_{kl}] - g_{M,k}$$
$$= -\frac{d}{dt} z_k = -\dot{z}_k, \;\; j \neq k$$

or

$$[\sum_{j=1}^{N} z_j(u_{kj}) \cdot [\mp\xi(j,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj}]$$
$$+ z_k(t) \cdot [\xi(k,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj}] + b_k [\sum_{l=1}^{N} \dot{\varepsilon}_{kl}(t) \cdot z_{kl}(t)] + g_{M,k}(t)$$
$$= \frac{d}{dt} z_k(t) = \dot{z}_k(t), \;\; j \neq k \;\; .$$

(12.20b)

Comparison with equation (12.13) shows the Richardson process reaction coefficients now are given by

$$a'_{M,ij}(u_{k,j}) = \mp\xi(j,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj} \quad , \; j \neq k \text{ (reaction coefficients);}$$

(12.21b)

that is, the same as case (a), except for the reversal of signs. By the same reasoning, in both cases the fatigue coefficients are found to be given by

$$-b_k = \xi(k,k) \Big/ \sum_{p=1}^{M+1} |v(p)|^2{}_{proj} \quad \text{(fatigue coefficient).}$$

(12.22)

The following restrictions finally apply:

Restriction (a).
From equation (12.11), $\alpha_k \neq \pm\pi/2$. Now introduce a refinement:

$$\pi/2 < \alpha_k < (3/2)\pi$$

The reason is as follows. Let v_k denote the space projection of velocity in $\bar{S}_k$ of the receiver k . Given that (f,v) is an inner product [equation (12.8)], we have $(f,v_k) = |f| \cdot |v_k| \cos\alpha_k$ which, combined with the above refinement, gives $(f,v_k) < 0$. The latter combined with the definition of ξ [preceding (12.11)] shows

$$\xi(k,k) < 0 \ .$$

(12.23)

[Recall α_k and α_{kk} mean the same thing; from the third of equations (12.3) $m(k) > 0$; and, from (12.2), $ds/dt > 0$.] This refinement on the prior restriction makes equation (12.22) consistent with the discussion following equation (5.10) whereby the second of equations (5.4), $b_k > 0$ is to be retained. The geometric meaning of (12.23) is that, *if we are speaking of a "normal" Richardson process, then in $\bar{S}_k$, the angle between the space projections of receiver velocity and force must be oblique or parallel, and the two projections must be oriented in opposite directions.* (That is, if one is in the 1st or 4th quadrants, the other must be in the 2nd or 3rd quadrants.)

Restriction (b) .
In addition, the sender and receiver (if they are distinct entities) must be separated by a finite spatial distance: Let x_{ik} and x_{ij} be the i^{th} spatial components in $\bar{S}_k$ of receiver and sender, respectively. Then the spacelike part of distance between them is given by $\varepsilon_{kj} = [\sum_i (x_{ik} - x_{ij})^2]^{1/2}$, by which $\dot{\varepsilon}_{kj} = \varepsilon_{kj}^{-1} \sum_i (\dot{x}_{ik} - \dot{x}_{ij})$ is undefined at $\varepsilon_{kj} = 0$.

[Note that, in the discussion of Restriction (a), the angle α_k is defined with respect to the receiver-specific frame $\bar{S}_k$. When these various frames are combined for all receivers k into a single reference frame, let us call it S_{ALL} , then the angle restrictions will be shifted to accommodate the relative orientations of the various $\bar{X}^1$ - axes within S_{ALL} . For example, consider a receiver $k = R$ that ends up situated on the axis X^1_{ALL} of S_{ALL} but oriented to "left" of the center of mass $\overline{cm}$ of the system. Anticipating Part 17, below, if we put the origin of the coordinate system at the $\overline{cm}$, then R will be situated on the negative part of X^1_{ALL} and the force "felt" by R will be the negative of that felt by a receiver in the conventional positive part of X^1_{ALL} . That is, in positioning R within S_{ALL} , we will have flipped its original $\bar{X}^1$-axis by $180^\circ = \pi$ (radians). The constraint corresponding to Restriction (a) applied to R then becomes $-\pi/2 < \alpha_k < \pi/2$.]

In sum, the above has shown that when the identifications given by (12.14), (12.15), (12.16), (12.21a), (12.21b), and (12.22) are made, then the generalized Richardson process, equation (7.10), is equivalent to a restricted case of equation (12.9), with the restrictions being those given as the preceding items (a) and (b).

This equivalence states that, as seen in any of the clock frames $\bar{S}_k$ (at clock time t ; i.e. as measured in physically real time), the magnitude of provocation emitted by any party l equals the magnitude of the space component of its velocity in $\bar{S}_k$; and the retarded time t' is the Richardson process provocation retarded time u_{kj} of the referent sender. [Note this latter assumption is a (more exact) restatement of the previous discussion in Part 9.] Equation (7.10) is a compound of time-varying reaction coefficients, time-varying grievance term, and time-varying fatigue coefficient; equation (12.9) is entirely a construct in the social space-time reference frame $\bar{S}_k$ associated with the party k . Thus the connection between the two is demonstrated.

[From this viewpoint we can revisit the composition assumption regarding the partial reaction functions, first introduced in Part 1. Given the above, this assumption corresponds exactly to the physical counterpart of sender-receiver relationships in electrodynamics: What a charged "reference" particle "sees" at a given moment and place is the *sum of all the separate electromagnetic signals*, each generated by a "source" particle (physics term ↔ sender party) at an earlier time, equaling the space distance from source position at time of emission to the present position of the reference particle, divided by the speed of light.]

Before leaving this section one more idea is in order; it is to modify equation (12.9) to include the effect of physical distance. Let $D_{jk}(t) > 0$ be the *geographic* separation of parties j and k at the time t . I include the functional dependence on time to reflect that societal entities do change their identities (change boundaries, merge with others, emerge from others, etc.) from time to time. For nation states, this distance could be the distance between national capitals (among other possibilities). Then make the change

$$(f_k, v_j, t) \to (f_k, v_j, t) \cdot D_{jk}(t)^W \quad , \tag{12.24}$$

where $W \equiv w(t) \geq 0$ is a declining non-negative function of time,

$$dw(t)/dt < 0 \ , \tag{12.25}$$

and has the approximate present value

$$w(t_P) \cong 0 \ . \tag{12.26}$$

From (12.24) , equation (12.9) is replaced by $(f_k, v_j, t) \cdot D_{jk}(t)^W \equiv f_i(k,t) \cdot v^i(j, t'_j)$, $t'_j < t$, from which

$$(f_k, v_j, t) = D_{jk}(t)^{-W} [f_i(k,t) \cdot v^i(j, t'_j)] \ , \ t'_j < t \ . \tag{12.27}$$

Let corresponding definitional changes apply to equation (12.8) and to the other forms of the linkage equation. The effect of this change is to make linkage values diminish with geographic distance between sender and receiver but, in view of (12.25), with decreasing sensitivity as time progresses; W thus acts like a loss-of-strength gradient (Boulding 1963). From (12.26), this gradient has declined to approximately zero by the present epoch, thus $D_{jk}(t_P)^{-w} \cong 1$, thus recovering equations (12.8), (12.9) and their related forms, where discussion is confined to present times. We will stay with the original (no loss of strength with distance) formulation of linkage until Part 17.

13. Networks Having a Nodal Structure.

Turning from Richardson processes, we need to become oriented, from the viewpoint of the space-time model, to a different aspect of the referent world of global relations. A putative key feature of relations between nations is their hierarchical character and, in particular, one aspect, which I will call nodal structure and which can be thought of as a topic in network theory (Buchanan, 2002; Newman, 2003). In brief, by the underlined phrase I mean a pattern in which most interactions among a collection of social units (in this instance nations) involve, repeatedly, one or more of the same relatively small group of such units as one or both of the interacting parties.

In greater elaboration, for many types of behavior, the pattern of interaction among nations may be approximated by what has been called "feudal structure" (Galtung, 1971, p.89). To simplify somewhat, the characteristics assumed of feudal structure are:

a) Nations belong to one of two disjoint groups, called the "center" and the "periphery," respectively;

b) Each peripheral nation interacts mainly with one and only one central nation;

c) No two peripheral nations interact;

d) Each central nation interacts with each of its "own" peripheral nations and with every other central nation. The central nations are the "nodes" of the nodal structure;

e) the peripheral nations are much more numerous than the central nations.

(13.1)

Below I shall refer to any party that is a member of the center as a central; any other party I shall call a peripheral.

Galtung may have chosen the word, "feudal", because the structure looks like the arrangement which one might imagine to have held in medieval Europe, whereby the inhabitants of each local domain conducted economic, religious, legal and other community relationships primarily with the ruling prince and his entourage rather than with each other or across community boundaries. (At the "center" was a castle. Is that the way it really was?) Galtung applies feudal structure to a context of meanings, viewed from the perspective of the participants, which he calls structural imperialism, by which he characterizes modern global society. Here, however, my interest is in the pattern, itself, divorced from any such application. It is to emphasize this point—that I only care about how the pattern looks, not what it means to the participants—that I will talk about nodal structure, in reference to the pattern only. Further, I will consider also as "nodal" certain modifications to the pattern just described; these are described in Part 14, below.

Available evidence indicates that nodal structure may be roughly accurate, in current times (perhaps post- Treaty of Westphalia, 1648?), for several kinds of interaction. The main apparent inaccuracies are, first, that most (or all) pairs of contemporary nations interact to some extent and, second, a fair number of peripheral nations interact to high degree with two and occasionally three central nations (instead of with just one). The factual descriptiveness of nodal structure is that the amount of pair-wise interaction is small, relative to total interaction, for a referent nation toward all but a few other nations. A figure appearing in Williamson (1985) indicates what may be a typical realistic case for data based on exports, foreign troops stationed on ones own territory, and arms transfers received from foreign sources. Referent nations which, in the late 1960s or early 1970s, concentrated the indicated actions on a particular other party are connected by lines, the arrows of which point to the other party. To recycle a couple terms, I shall refer to the other party as the sender and to the given party as the receiver. This wording anticipates that these nodal roles will be seen to *correspond to the previously introduced Richardson process roles of the same names*. Similar data for total directed trade (sum of trade from each party to the other) between nation dyads are depicted in Figure 13.1, for which the data were taken from Barbieri (1996, 1998a, 1998b). (Table 13.1, which follows Figure 13.1 after the end of the text pages, identifies the nation numbers used in the figure.)

The directed connection between a receiver and a sender—the line and its arrow—I will call the dichotomous linkage. Arrows are omitted where the other unit is the focus of many such lines. (Though not applicable to the data used here, note that the physical direction of action may be from receiver to sender in the case of one-directional exports but is from sender to receiver for troop deployments and arms transfers.)

The specific criterion for connecting two nations in Figure 13.1 was that 20% or more of the total behavior of the particular type (in this instance total trade, but it could also be one-way trade, troop deployments or arms transfers) involving the receiver be to or from the given sender. Shortly, a different linkage criterion will be given. Note that interactions of high volume will not result in a connection in the figure if each bilateral value is a small fraction of receiver's total. This works to reduce the number of linkages in which centrals are receivers (linked either to other centrals or to peripherals), because, in such instances, most or all bilateral values are small fractions of the respective party's total. An effect of these procedures is to make peripheral parties usually the "receivers", relative to United States, Japan, Russia, etc. as "senders", at or above the 20% criterion for total directed trade, as in Figure 13.1; and for exports, arms transfers, and military personnel deployments, as indicated elsewhere (the latter using a 25% criterion; Williamson, 1985 and 1989).

14. Alternative Definitions of Continuous Linkage.

In the above formulation, linkage is a discrete variable with two possible states: either one party is linked to another or it is not. Now consider a reformulation, from dichotomous to continuous variable, in which large positive values correspond to the linked cases in the discrete sense of linkage. There are several ways of doing this; which is preferable is unclear at this juncture. Two of them, discussed below, involve computing a norm of interaction volume (possibly an expected amount, and based only on the identity of the receiver) then, for each named sender, subtracting that value from a function of the observed receiver-sender interaction value. These alternative linkage measures are sketched below:

In the first alternative, the linear linkage variant, the norm is the arithmetic mean of receiver interactions to all senders. The function, which involves the observed interaction value itself, is expressed as

$$\theta_{lin}(j,k) \equiv w(j,k) - \sum_{l=1}^{N} w(j,l)/N \tag{14.1}$$

where $w(j,l)$ denotes the interaction behavior involving (in either direction) receiver j and sender k (and summation on senders l). In a second, the logarithmically linear variant, the norm is the geometric mean $\langle x_j \rangle$ of non-zero receiver interactions to all senders. The function is

$$\theta_{\log-l}(j,k) \equiv \ln(w_{j,k} / \langle x_j \rangle) = \ln(w_{jk}) - \ln \langle x_j \rangle \ . \tag{14.2}$$

Note that, in both expressions, the first position within the left hand parentheses refers to the receiver, the second position to the sender. I will observe this convention throughout the discussion.

Whether linear, logarithmic, or some other variant best defines linkage, the important point here is that the continuous linkage value be able to assume positive, negative, or zero values. This ability characterizes both equations (14.1) and (14.2). The case of a positive value will be referred to as a state of linkage from sender to receiver. The case of a negative value will be referred to as a state of alienation of sender from receiver.

The two linkage measures are illustrated in figures 14.1 and 14.2, using total directed trade data (Barbieri, 1998b). In these figures, linkage values are calculated for United States and Russia as senders, for the year 1998, and scatter plotted against each other. Below [Part 17, following equation (17.11)] we will consider the reason for the particular choice of parties and year. For a reason to be given (also in Part 17), the stronger correlation is of greater interest; this points to the log-linear linkage measure (r = - 0.42) of Figure 14.2, corresponding to equation (14.2). The fewer number of cases available in the log-linear case, however (N = 81 versus N = 136) is a disadvantage. This of course arises because of numerous cases of zero bilateral trade values, particularly involving Russia as sender. A conceivable remedy is to treat the zero-valued cases as missing data, to be estimated by some supplementary (and probably ad hoc) procedure but, at present this is merely conjecture. In sum, both possible candidate operational empirical continuous linkage measures have drawbacks that seem to call for some third, yet to be realized hybrid; but either one will suffice for the present illustrative purpose. What follows in this discussion omits choosing among them.

With the aid of the continuous linkage formulation, three of the feudal structural characteristics, the expressions labeled (13.1) in Part 13, above, can be modified as follows:

> b') As a sender, each peripheral nation has a negligible θ-value with any central; as receiver, each peripheral may have non-negligible positive or negative θ-values with one or more central nations.
>
> c') As sender or as receiver, each peripheral nation has negligible θ-values with other peripherals.
>
> d') As sender, each central nation may have non-negligible positive or negative θ-values with one or more peripheral nations; as receiver, each central has negligible θ-value with any other nation.

(14.3)

In what follows, expressions (13.1), as modified by (14.3) will be taken as the definition of a nodal network.

15. Nodal Network Compared with the Richardson Process.

Recalling the space-time formulation of Richardson processes (Part 12), in the clock frame $\overline{S}_k$ an inner product between a sender ℓ velocity and a receiver k force was defined by

$$(f,v) \equiv f_i(k,s)\cdot v^i(\ell,k,s'_\ell)_{proj} \ , \quad s'_\ell < s \ . \tag{12.8}$$

Given the functional dependence of s on t, the special orientation defined for $\overline{S}_k$, the identification of social field with Richardson variables given by equations (12.14) through (12.22), and the restrictions labeled (a) and (b) following (12.22), equation (12.8) then becomes equivalent to the generalized Richardson process— the latter a function of time-varying reaction coefficients, time-varying grievance term, and time-varying fatigue coefficient. Putting this conclusion another way, in the space-time continuum the Richardson process appears in the guise of (or is represented by, or isomorphic to) a special case of the inner product (f,v). Let us call this inner product the R-process function.

Now we want to consider a second type of inner product. To construct it we need to consider again the (spacelike) force defined above in equations (12.3). First, we will assume that any such spacelike force felt by a given referent party includes contributions from each of the other parties in the global system—the sources, with each such contribution in the form of a force vector term (possibly with vanishing components) to be added to the others to form the net spacelike force vector acting on the receiver. (In the physical analogue, this net receiver force possibly may also include additional contributions from the electromagnetic field but this possibility is not used here.) Now suppose that a receiver R (the referent party) happens to be at rest relative to some reference frame S' . Then, recalling equation (12.6), any one of the above force terms that R feels from any other party will have vanishing time component, thus lie entirely in the R space sub-continuum relative to S' . [This follows because (12.6) must remain true even if the other party is the only other party or other contributing source in the system, thus the only contributor to the force felt by R .]

In what follows let us further assume that any such spacelike force term, the contribution of a specific other party, is oriented on a line passing through R and the retarded space position of the other (source) party, in the special case where the other party is at rest or moving toward or away from the given referent party, relative to S' . Let us call this the orientation assumption. With this combination of assumptions and special circumstance one can classify a particular spacelike force as attractive—meaning the direction of the force vector on the referent party is toward the party exerting the force; or repulsive— meaning the direction of the force vector on the referent party is away from the party exerting the force. If the special at-rest or relative source motion condition does not hold, the attractive or repulsive trait is that which would apply, if in fact it did hold. (We require that this condition always be meaningful, which it is, given the physical exemplars that are being emulated.)

Next we assume that the societal spacelike force that we have been considering is composed of an attractive force f_α^A and a repulsive force f_α^R :

$$f_\alpha = f_\alpha^A + f_\alpha^R \ , \tag{15.1}$$

when written for an entirely general reference frame S . We assume that f_α^R is spacelike and that

$$v^\alpha f_\alpha^R = 0 \ . \tag{15.1a}$$

[Compare this with equation (12.5). For the attractive force f_α^A the orientation assumption can be derived from equation (4-101), Rohrlich, 1990, p.85.] Each of these two force terms is to be regarded as the sum of all the separate contributions from other parties, as described in the preceding paragraph. In the frame $\overline{S}_k$, equations (12.8) and (12.9a) then become

$$(f,v) = f_i^A(k,s)\cdot v^i(j,s_j') + f_i^R(k,s)\cdot v^i(j,s_j') \ , \tag{15.2}$$

and

$$(f,v) = f_A^1(k,t)\cdot v^1(j,t_j') + f_R^1(k,t)\cdot v^1(j,t_j') \ . \tag{15.3}$$

In (15.3) and similar equations, the tags A and R have been tucked into the available spaces in the subscript or superscript positions; the meanings are the same as the superscripts in (15.1) and (15.2). Note, also, that the constraints on the relative time order, s_j', t_j' versus s,t , respectively, have been removed. Next define

$$\theta(j,k,t'_j) \equiv (f,v) - f_i^{\,A}(k,t)\cdot v^i(j,t'_j)\ ,\ t'_j > t\ , \tag{15.4}$$

where now the constraint of relative time order is the reverse of what it was in Part 12, and the new quantity θ is again defined at a later time t'_j but this time is now specific to the party having the named velocity rather than to the party having the named force, as before. From (15.4) and (15.2) we find

$$\theta(j,k,t'_j) = f_i^{\,R}(k,t)\cdot v^i(j,t'_j)\ ,\ t'_j > t\ ; \tag{15.5}$$

that is, $\theta(j,k,t'_j)$ is the inner product formed just by the repulsive term of the social force. This idea of exchanging the time order between velocity and forces allows us to consider a party—one of the “senders” in our prior discussion—who receives information on the force felt at a *previous* time (rather than at a later time, as before) by another party—the “receiver” in our prior discussion. So now it is information about the force on a party that is being received by another party at a later time. (Of course, both types of sending / receiving are happening to all parties at all moments.)

At this point we need to exercise some care in establishing further use of terminology: In what follows I will continue to use “sender” to label the party emitting a signal and “receiver” to label the party receiving a signal. In this new context of equation (15.5) and equivalent expressions, the receiver (rather than the sender, as before with the R-process function) is now providing the velocity vector and the sender (rather than the receiver, as before) is now providing the force vector. (Thus we have the odd seeming consequence that the R-process—like “response” precedes the “provocation”; although, of course, we are no longer speaking of the normal sense of an R-process, rather we are speaking of linkage, and the various terms no longer have precisely their original applications.) We will see the reason for this exchange of roles later on. Let us call this new inner product the linkage function. Shortly we will connect this function to the idea of linkage first encountered in Part 13, above.

Why this exchange in time order of roles? First, the previous discussion reflected the idea that the force on, thus acceleration of, a party (corresponding to what in Richardson’s original equations were his “ dx/dt ” and “ dy/dt ” terms) are caused by, thus follow in time, the (earlier) velocities of other parties (his original “ x ” and “ y ” terms); as previously developed this corresponds to the condition $t'_j < t$. Second, the discussion to follow will reflect the idea that the condition of “linkage”, which below will be related to the velocity of a party, is caused by, thus follows in time, the (earlier) forces on other parties, which corresponds to the condition $t'_j > t$. [Note this time order corresponds, empirically, to the observed lag time in achievement (e.g. USA) and in loss (e.g. UK, France) of major power status by nations, relative to their internal material capabilities—which is part of the reason for the altered time order.] In sum, the linkage function involves the repulsive force term only and the time order between velocity and force information is reversed, relative to what it was for the R-process function.

Now, for a given receiver k , let $\bar{S}_k^R$ denote a new reference frame in which the X^1- axis has been chosen parallel to the space sub-continuum projection of the repulsive force f_α^R acting on the sender (rather than parallel to the net force, as in $\bar{S}_k$). Thus all other spacelike components of f_α^R vanish, $f_{i,k}^R = 0$, $i > 1$. Let $\bar{v}_R^i$ denote the receiver velocity spacelike components in $\bar{S}_k^R$.

Next consider the cross-tabulation shown in Table 15.1 . As shown, the non-zero θ values are confined to the cell corresponding to non-vanishing components $i = 1$ in each of the force and velocity vectors. (One can also generalize the key conclusion: in any clock frame S , for θ to be non-zero there must be at least one value of i for which $f_i^R \neq 0$ and $v^i \neq 0$.) Referring now to Figure 14.2, if we can arrange for the extreme linkage observation values—those appearing in the upper left and lower right of second and fourth quadrants—to correspond to the case appearing in the lower right-hand cell of Table 15.1, and for all other observation values to correspond to one of the three other cells, then equation (15.1) will have *approximately* replicated the observed linkage data. (Approximate because the realistic contrast in $\bar{v}_R^1$ values is "small" versus "large", rather than the zero versus non-zero of Table 15.1 .) This we further consider in Part 16, below.

We can also generalize the above discussion to include the modification for geographic distance (in times before the current epoch) indicated in equations (12.24) through (12.27). This is done by inserting the factor $D_{jk}(t)^W$ as a multiplier of each occurrence of (f, v) and $\theta(j, k, t_j')$ in equations (15.2) through (15.5). For instance, equation (15.5) becomes

$$\theta(j,k,t_j') = D_{jk}(t)^{-W} \cdot [\, f_i^R(k,t) \cdot v^i(j,t_j')] \ , \ t_j' > t \ . \tag{15.6}$$

The previous conclusions are unchanged.

Table 15.1 . Linkage function values in $\bar{S}_k$ under idealized circumstances.		
	velocity components:	
repulsive force component:	$\bar{v}_R^1(j,t_j') = 0$	$\bar{v}_R^1(j,t_j') \neq 0$
$\bar{f}_1^R(k,t) = 0$	$\theta(j,k,t_j') = 0$	$\theta(j,k,t_j') = 0$
$\bar{f}_1^R(k,t) \neq 0$	$\theta(j,k,t_j') = 0$	$\theta(j,k,t_j') \neq 0$

Supposing, for the moment, that the replication indicated by Table 15.1 has been accomplished, let us (further) compare R-process and linkage functions. One difference between them is that the R-process function involves the complete force acting on the receiver, while the linkage function force (acting now on the sender) involves only the repulsive term of that force. Their commonality is that (1) both are constructed from inner products involving velocity of one party and forces acting on another party, and (2) both involve delayed receipt of the signals that we have been calling provocations (though the velocity-force order is interchanged). Put another way, Richardson process and linkage relationships *differ only in the relevant part of the force acting on one of the parties*, plus the *time order of occurrence* of the signaling events that we have associated with force (and acceleration), versus velocity.

The reader may object that whatever is causing, say trade, between one party and another is certainly not the same as a “provocation” in the arms race sense. Fair enough, that distinction may (or may not) be correct; but what I propose is that, for both R-process and linkage functions, the reference to signals be reinterpreted and confined to that which one might assign to a behavior such as trade, to the extent that such behavior differs from arms racing behavior. Thus equation (12.13) and the several like Richardson equations preceding, starting with equation (5.5), are to be reinterpreted as describing a process like an arms race but involving a “race” in terms of nominally cooperative behaviors. [Of interest is that the original use of the reaction curves described in Part 3 was as a description of competing parties in an economic duopoly (McGuire 1965).] This reinterpretation is a suggestion that a process like an arms race is happening in cooperative activities like economic trade and that this process is *precisely* related to another similar process that we recognize as the nodal structural pattern in global interactions like those cited in Part 13. Success in using the linkage function to replicate empirical data on linkages would be evidence in favor of such a similarity.

Recall that the linkage function involves a present receiver velocity and a retarded sender force. Just as for the R-process function, in space-time model terms this delay reflects the finite signal propagation speed. In the terms with which we began this discussion, we might regard the delay as the product of the time required for calculations by the parties, expressed as the utilities introduced in Part 2, to result in the reaction, grievance, and fatigue terms leading to Part 5 and expressed above as the value of the linkage function. That is, the linkages and supporting preferences and calculations follow the forces (which, we have seen, are expressed as the Richardson process rates of change). In space-time terms, first A signals B (and other parties C) via a velocity that B later “sees”; which is the R-process part. Then B signals A (and other parties C) via a force that A “sees” still later, which is the linkage part. These two parts are happening at all times between all pairs of parties. Since social inquiry conventionally regards preference and calculation to precede behavior, in conformity with that convention let us attribute the calculations always to the receiver. (To reiterate a previous point, for R-process this is the party feeling a force thus exhibiting acceleration; for linkage it is the party exhibiting the velocity; the two roles are exchanged.) Let us also recognize the possibility that this attribution convention may be nominal.

Now I need to make clear an important point. This is the qualification to which I referred, following equation (12.3) above. Both the R-process quantity (f,v) , introduced in equation (12.8), and the linkage quantity $\theta(j,k)$, introduced in equations (14.1), (14.2), (15.4), and (15.5), involve the construct of "force" or various parts of it, as in equation (15.1). These uses of "force" are purely "phenomenological" in the sense that none of them are to be regarded as constructively giving the numeric values of the components of force. Rather, force is regarded to arise from separate considerations, modeled after modifications of Lorentz and Yukawa physical forces which are not actually discussed in the paper except in a very cursory way. This means that the societal forces are to be constructively defined **a priori** using these analogues, from which the linkages and R-process quantities (reaction, fatigue, grievance) are then to be calculated; in turn, the utilities with which we began this discussion are to be inferred from the latter (plus further constraints such as a parabolic form to the utility contours). While any particular discussion (e.g. this one) of such concepts might precede in a different order, should discrepancies arise the above is controlling. Thus the explanatory ("causal") sequence is the reverse of customary social theorizing, which latter begins with "motives", i.e. with utilities.

16. Linkage in Relation to Other Phenomena.

We have four remaining empirically referent topics to connect to the space-time model, which we do in this section. In each of three sub-sections dealing with these topics we end by posing one or more issues concerning them, to which we return, starting in Part 17.

Civilizational convergence (Wilkinson 1987) refers to the idea that, beginning ~ 5,000 to 10,000 years before the present, (a) numerous geographically local, initially (partly or wholly) isolated human civilizations arose; (b) over the period of time ending in the present, these civilizations gradually merged into the one global civilization that characterizes human organization at present.

National power. A recurrent issue in world politics study concerns national political "power". To begin, what is it? One point of view treats political power as equivalent to national material capabilities; for instance, political power may be equated to (made empirically operational as) national gross domestic product, or to the Correlates of War project material capabilities index (an average across 6 indicators each expressed as fraction of respective world total), etc. An alternative point of view regards power to be related to, but distinct from, national material capabilities. Capabilities may be translated to power; to what extent this happens depends on other factors in addition to material capabilities. Neither viewpoint is entirely satisfactory: the first point of view is not entirely convincing to all who face the issue. The second begs the question how, *quantitatively*, does one think of the translation from capabilities to power; how, *quantitatively*, is the one different from the other? A related question concerns the "efficiency" of the translation: what does it mean to speak of the efficiency with which capabilities are translated into political power?

Spheres of influence -- bimodal polarization. The latter can be regarded as a special case of the former; namely two spheres of influence. The oft cited example is the "East-West" polarization of the Cold War period, 1945 to c. 1991. Can anything of a quantitative nature be added to the considerable literature (much of it quantitative) defining and studying polarization and spheres of influence?

Power transition. [See, among others Houweling and Siccama (1990), Kugler, Lemke and Tammen (2000), Lemke (2002), Organski (1968), Organski and Kugler (1980), Wayman (1983) and (1989).] The aspects of note from this concept are, first, that, upon the occasion of such a transition, several things are said to happen concurrently or in sequence: (1) rapid internal "development" of a "challenger" nation is juxtaposed against the pre-existing national power of a "status quo" nation; (2) the probability of global war is substantially increased; (3) upon the conclusion of such a global war, if any, there may be a radical rearrangement of international alignments; (4) the putative challenger, even when suffering defeat, may (if it survives) subsequently recover its internal capabilities and its political power. (The disintegration of various empires after the First World War and, even more strongly, the emergence of nuclear armaments prompt the qualification in item 4.) The second aspect is that the international alignments of item 3 are, themselves, thought to have arisen because one nation or a handful of them completed the process of development before all others; and the challenger comes from the ranks of the more recently developed. Again the question is what, quantitatively, can be added to existing discussion of these putative features of the global system?

17. Postulates and Implications of System Dynamics.

To recapitulate: the above has led to the questions how plausibly to generate the nodal pattern developed, in Parts 1 through 15, and whether the additional issues in Part 16 can be further developed. In answer, I postulate a set of global system dynamic conditions, formulated in terms of the space-time framework. In the Part 19 we touch on the possibility of computational space-time model dynamic simulation as a means of studying the global system. Suppose the qualitative statements shown in Table 17.1 (next page).

[Why choose the particular configuration—two forces, one global and attractive, the other local and repulsive, and the other elements in this table? The short answer is, the indicated configuration achieves the desired result: it appears able to represent the various empirical aspects discussed above and below. (I say "appears" because the present reasoning is impressionistic and qualitative; the more adequate reasoning probably entails working a numeric computational model, yet to be written. See Part 19, below.)]

Now I will show how the characteristics postulated in Table 17.1 connect to the idealized linkage values in Table 15.1, and to the other phenomena mentioned in Part 16. Numbers in double brackets, "[[...]]", refer to the corresponding items in Table 17.1 .

17.1. Results concerning linkage functional values—the nodal network.

Consider two parties j and k . From [[7]] and [[11]], in the current epoch each party either is in group L (feels the repulsive force) or is not. If we identify group L as the group of centrals then the remaining parties constitute the peripherals. Thus there are only four possible cases:

1. j is peripheral, k is central;
2. k is peripheral, j is central;
3. both j and k are peripheral;
4. both are central.

(17.1)

Case 1: k is central, thus from [[7]] it feels a repulsive force $f_\alpha^R(k) \neq 0$. In the clock frame $\bar{S}$ this becomes

$$\bar{f}_\alpha^R(k) \neq 0 \ . \tag{17.2}$$

From [[8]] the space velocity components are constrained by $\bar{v}^i(k) \cong 0$, thus from equation (15.1a)

$$\bar{f}_0^R(k) \cong 0 \ . \tag{17.3}$$

[like equation (12.6) but, in this instance referred to $\bar{S}^R$]. From (17.2) and (17.3) we find

$$\bar{f}_i^R(k) \neq 0 \ . \tag{17.4}$$

Text continues page after next.

Table 17.1 . Posited global system dynamic characteristics.
I. All epochs, all parties (see Part 15, above). Each ...
[[1]] ... exerts attractive force f_α^A on *each of the others*; i.e. f_α^A is "*global*".
[[2]] ... exerts repulsive force f_α^R on *others at sufficiently small distances, only*; i.e. f_α^R is "*local*" .
[[3]] ... possesses a <u>charge</u> q (analogue of electronic charge); exerts/experiences forces f_α^A , f_α^R on/from other parties in proportion to magnitude of q .
[[4]] ... is characterized by linkage as modified by geographic distance, equations (12.24) through (12.27) and (15.6).
[[5]] ... otherwise, emulates dynamics of electrically charged particles.
II. "Current" epoch (~ year 2000).
Group L ("local") nations:
[[6]] Relatively few in number.
[[7]] Mutual space distances sufficiently small to feel mutual f_α^R forces (from each other).
[[8]] Typical $[(d\overline{x}_i / dt)(d\overline{x}^i / dt)]^{1/2} << c = 1$ (magnitude of Newtonian velocity space components small in clock frame $\overline{S}$).
[[9]] Clustered about system center-of-mass $\overline{cm}$.
Other (non-L) nations:
[[10]] Relatively numerous.
[[11]] Mutually remote (space distances from most others too large to feel f_α^R forces).
[[12]] Typical $[(d\overline{x}_i / dt)(d\overline{x}^i / dt)]^{1/2}$ non-negligible fraction of 1 (magnitude of Newtonian velocity space components large in clock frame $\overline{S}$).
[[13]] Typical $\overline{v}^i$ shows large component toward or away from system center of mass $\overline{cm}$.
III. "Primitive" epoch (~ 5,000 to 10,000 years before present).
Group L ("local") nations:
[[14]] Non-existent.
Other (non-L) nations:
[[15]] Characterized by [[8]] and [[11]], above.

Text continues here:

In addition, j is peripheral, therefore [[12]] applies. We consider two sub-cases.

(a) The first is that $\bar{v}_R^1(j,t_j')$, $t_j' > t$ does not contribute to $(\bar{v}_i\bar{v}^i)^{1/2}$; i.e. the space part of velocity does not involve component 1 which, recall, was chosen parallel to the space component of the replusive force acting on the central k ; that is $\bar{v}_R^1(j,t_j') = 0$, by which $\theta(j,k,t_j') = 0$, case 1a.

(b)The alternative possibility is that $\bar{v}_R^1(j,t_j') \neq 0$, as would hold if $\bar{v}^i(j,t_j')$ and $\bar{f}_i(k,t)$ were approximately parallel or anti-parallel. This, together with (17.4) puts case 1 in the lower right-hand cell of Table 15.1, from which we conclude $\theta(j,k,t_j') \neq 0$, case 1b. Thus the central k and the peripheral j, the first as sender, the second as receiver *may*, but need not, be linked.

Cases 2 and 3. In both instances k is peripheral thus, from [[11]], we have (in most cases) $f_\alpha^R(k) = 0$. Reference to Table 15.1 then shows $\theta(j,k,t_j') = 0$, cases 2 and 3.

Case 4: Both j and k are central thus, from [[8]], we have $\bar{v}^i(j) \cong 0$ (in any clock frame $\bar{S}$. This, equation (17.4) , and reference to Table 15.1 show $\theta(j,k,t_j') \cong 0$, case 4.

In addition, to achieve a pattern that is “nodal” as in Figure 13.1 requires postulates [[6]] and [[10]] as separate conditions. Implicitly, these two postulates are reflecting the juncture to which the global system has evolved (see discussion below, of convergence) in the current epoch, in which relatively few centrals have yet emerged.

In sum, case 1, in which receiver is peripheral and sender is central is the only combination yielding (1b) non-zero linkage values, though it may yield zero (1a) values as well. The cases of type (1b) comprise most of the dichotomous linkage connections in Figure 13.1, provided we regard the centrals to consist of France, Japan, United Kingdom, USA, and (a poor fifth) Russia. (See the comments in Part 17.2, below, on the position of Russia using other linkage indicators.) The logic is not a perfect fit to the indicated data—the imperfections consist of approximate rather than exact equalities in various places in the argument. (For example, a sufficiently large force could couple with a small but non-zero velocity to produce a small linkage, say, between two centrals.) Nor have I empirically tested, in any systematic way, these ideas. (Such tests probably await development of a computational version of the dynamic modeling ideas presented in the present work.) However the similarity of the above to figures 13.1, 14.1, and 14.2 is suggestive.

17.2. Results concerning other phenomena.

Turning to the other ideas mentioned in Part 16, above, civilizational convergence can be fit to the following reasonable conjecture regarding the nodal structure. The conjecture is that nodal structure is a continuing feature of human history, going back to the beginning of human civilization, ~ 5,000 to 10,000 years before the present: As one moves backward in time, the linkages of nodal structure gradually weaken, the structure itself becomes fragmented into several pieces, each a smaller nodal structure isolated from the others. (This possibility is the reason for the qualification “most others” in postulate [[11]] of Table 17.1.) China, America, India, Europe-Middle East, and other culturally distinct regions, are plausible candidates for such isolated structures. (For a more complete list see Wilkinson 1987.) Earlier than the agricultural period of human experience—say earlier than ~10,000 years before the present—we may suppose that the linkages disappear entirely as a description of relationships among human groupings; except perhaps that one can think of nodal structure as merging, in the prehistoric past, with the hierarchical authority patterns of tribal or extended family units. Going forward in time, this assumption says that nodal structures began to form (or to emerge as patterns distinct from tribal or familial organization) at the beginning of human civilization, manifested themselves in the several regional agricultural empires, then finally coalesced into a single globally extended pattern in the modern era.

The suggested connection to civilizational convergence is that it corresponds to / can be represented by this coalescing of nodal patterns. The corresponding feature desired of a dynamic space-time model is that the latter be able to exhibit such a pattern of coalescence. Two questions thus arise. First, what is the distant antecedent of the contemporary nodal pattern; second, how did the system get from that earlier pattern to the present one. In Table 17.1, postulates [[14]] and [[15]] address the first question; in fact, they speak of the prehistoric time before any regional civilizations had arisen that I have conjectured in the preceding paragraph.

To address the second question, consider postulates [[3]] and [[5]]. A feature of electrodynamics is that when electrically charged particles experience acceleration (change in magnitude or direction of velocity) they radiate energy; thus, due to conservation of energy, the particles lose energy-momentum. (Relativistically, the hyphenated pair are a single vector entity.) This feature is carried over into the suggested social space-time analogue of a charged particle, the societal unit (party, agent), because it is intrinsic to the mathematics, which I have here proposed be adopted with slight changes. (The changes are: First, implicit in [[1]], like charges attract rather than repel; second, the local force postulated in [[2]] has no counterpart in electrodynamics, though it is patterned after the attractive—not repulsive—Yukawa potential in nuclear physics. That these changes do not alter the property of radiating energy follows from the fact that, in the physical exemplar, an electrically charged particle radiates in response to acceleration caused by any force, regardless of its mathematical character. Putting it another way, the mathematics involved do not “care” whether they are discussing electrically charged particles or social units.)

Loss of energy-momentum by the social units is a key feature. (My thanks go to Craig Chernos, my former student, for this idea.) Without it, there could be no secular evolution of the system: Under the attractive and repulsive forces, pairs of parties would come together, then fly apart, then come together again, ... in a perpetual yin and yang of kinetic energy converted to potential, then back to kinetic, etc. The process would be complex (with many interacting parties) but at root cyclical and perpetual, since there could be no loss of energy. The radiation of energy prevents this by insuring a continual loss of energy from the system of social units (i.e. an open system); thus the parties gradually approach an equilibrium in which the system is static because no available energy remains to be lost. In this model the current world is at some unknown distance from that equilibrium but, perhaps, nearing it. (The "end of history"?)

Now take a look at postulates [[1]] and [[15]]; in combination they imply that the prehistoric “cloud” of (non-central) parties drifted toward the system $\overline{cm}$. After sufficient time, some of these parties came within sufficient proximity of each other to feel the mutually repulsive force f_α^R . This generated the possibility for those parties to act as centrals, i.e. as the foci of linkages, which was realized. In view of the distance effect to which postulate [[4]] refers, however, in early times the resulting nodal structure was limited to geographically nearby social units. This result corresponds to the geographically limited civilizations of ancient times noted in Wilkinson's (1987) scheme. As the parties were subjected to forces (attractive and repulsive), according to the third of equations (12.3) concurrently they experienced acceleration; consequently they radiated energy-momentum. This process gradually allowed geographically separate groups of centrals (local civilizations), if they survived, after a time to come together—via mutual global attraction (f_α^A), local repulsion (f_α^R), and radiation of energy-momentum (as discussed above)—to form the single group of centrals in the system as it is now, in the current epoch. In this manner the postulates appear to get us from the distant pre-historic past to the present and to civilizational convergence. ("Appear", because resolution of the question actually calls for numeric computational simulation.)

National power is accommodated as follows. First, let us look in more detail at how the attractive force on party k at time t is determined. Following postulates [[3]] and [[5]], f_α^A is to be treated exactly as the (mutually repulsive for like charges, but otherwise identical) force of electrodynamics. The transcription of that force into our societal model terms can be summarized as follows. As seen in any clock frame $\bar{S}$, for k as receiver, and senders j

$$\bar{f}_A^{\,\mu}(t) = q \cdot g_A^{\,\mu}[\bar{v}_\alpha(t), q(j), \bar{v}^{\mu}(j,t'_j), \bar{v}^{\alpha}(j,t'_j), \Delta x_{sig}^{\mu}(j,t'_j), \Delta x_{sig}^{\alpha}(j,t'_j), \rho(j,t'_j)] \ ,$$

where
$j = 1,2,...k-1, k+1,...N$, $j \neq k$;
q is charge on the receiver k ; $q(j)$ is charge on senders j ;
$\bar{v}_\alpha(t) \equiv$ velocity components of k ; $\bar{v}^{\mu}(j,t'_j)$, $\bar{v}^{\alpha}(j,t'_j) \equiv$ velocity components of senders j ;
$\Delta x_{sig}^{\mu}(j,t'_j)$, $\Delta x_{sig}^{\alpha}(j,t'_j)$ $\equiv$ signal displacements from senders j to receiver k ;
$\rho(j,t'_j) \equiv u_\eta(j,t'_j)\Delta x_{sig}^{\eta}(j,t'_j)$, $u_\eta(j)$ $\equiv$ unit vector orthogonal to $\bar{v}^{\eta}(j,t'_j)$;
and $t'_j < t$, t'_j the moment at which signal received at t was emitted by j .

(17.5)

[The above is based on discussion found in Rohrlich (1990), pp. 85, 188, 191 and 192.] The repulsive force is treated in a similar manner but omitting most of the variables named in equation (17.5). It can be summarized by

$$\bar{f}_R^{\,\mu} = q \cdot g_R^{\,\mu}[q(j), \rho(j,t'_j)] \ , \tag{17.6}$$

where the constraints are the same as for (17.5).

[Note that both (17.5) and (17.6) have the same time order as the R-process: It is sender velocity information from a preceding time that later is viewed by the receiver. In addition to attractive and repulsive properties that are the opposite of their physical exemplars, f_A^{μ} and f_R^{μ} also arise from the same charge q for both, whereas physical charges differ in electromagnetic versus strong nuclear forces. Further, f_A^{μ} and f_R^{μ} implicitly result from vector versus scalar potentials, respectively. The (very much inconclusive) reason for this is to stay close to the physical exemplars at this juncture.]

Abbreviating the right-hand sides of expressions (17.5) and (17.6) to $q \cdot g_A^\mu$ and $q \cdot g_R^\mu$, respectively, the total (spacelike) force on receiver k becomes

$$\begin{aligned}\bar{f}^{\mu} &= \bar{f}_A^{\mu} + \bar{f}_R^{\mu} \\ &= q(g_A^{\mu} + g_R^{\mu})\end{aligned} \tag{17.7}$$

and the repulsive force becomes

$$\bar{f}_R^{\mu} = q \cdot g_R^{\mu} \ . \tag{17.8}$$

In addition, see Part 12, there is a "radiation reaction" force due to the loss of energy-momentum as the party experiences acceleration [Rohrlich, 1990, pp. 137-145, especially equation (6-57)]. See also the above discussion of civilizational convergence.

The significant conclusion from (17.7) and (17.8) is: each component of the force on the receiver is proportional to its charge q . Now let q be regarded as *representing the "internal" power of party* k . *If the party is a nation, then its charge is the social space-time representation of its internal national power*.

From (15.6) and (17.8) we find, for sender k

$$\begin{aligned}\theta(j,k,t) &= D_{jk}(t)^{-W} \cdot q(k) \cdot [g_R^1(k,t) \cdot \bar{v}^1(j,t'_j)] \text{ , frame } \bar{S}_k \text{ ,} \\ &= D_{jk}(t)^{-W} \cdot q(k) \cdot [g_i^R(k,t) \cdot v^i(j,t'_j)] \text{ , frames } \bar{S} \text{ , } t'_j > t \text{ ,}\end{aligned} \tag{17.9}$$

where on this occasion the time order is that which is appropriate for the linkage function. [So, if t''_ℓ are the times at which parties ℓ emitted the signals contributing to the repulsive force felt by party k at time t , as in equation (17.6), then the time order for the whole sequence is $t'_j > t > t''_\ell$.] The first line in (17.9) is expressed relative to the clock frame $\bar{S}_k$ of receiver k ; the second is valid for any of the clock frames $\bar{S}$ [which, recall from Part 8, and discussion following equation (8.1), have the same time axis as $\bar{S}_k$ but may have different space axes].

There is another way of expressing an inner product between two purely space vectors [Euclidean sub-continuum k -components, equation (11.8)]: It equals the product of their vector magnitudes $\times$ cosine of the acute angle between the velocity vector and the negative of the force vector. Using this formulation the second of (17.9) can be expressed as

$$\theta(j,k,t) = -D_{jk}(t)^{-W} \cdot q(k) \cdot \cos(\omega) \cdot |g_R| \cdot |v(j,t'_j)| \text{ , frame } \bar{S} \text{ ,} t'_j > t \ . \tag{17.10}$$

[The negative arises because the central local force is repulsive, thus it points away from the system $\overline{cm}$ whereas, if we put a peripheral on the same side of the $\overline{cm}$ as a central to which it is positively linked, then the peripheral velocity will point toward the $\overline{cm}$; thus in (17.10) the negative sign correctly orients the velocity direction.]

The above tells us that linkage involving any peripheral receiver is proportional to the central sender charge, which is identified above with the internal power (material capabilities) of the sending party. This fits well the role of θ as a reflection of the extent to which the central sender monopolizes [or, in case of alienation (negative θ), repels] the attention of the receiver:

> *Internal power translates into the ability to monopolize or repel behavior from others or, put in a more customary formulation, the ability to influence behavior.*

At the same time, the other factors on the right-hand side of (17.10) fit well the idea that

> internal power / material capability is not to be *equated* with influence; the latter is also mediated by other considerations.

Equation (17.10) tells us that, within the societal space-time representation, those other considerations are to be encoded by:

> ω , relative orientation of peripheral receiver velocity and central sender local force—which below will be equated with peripheral membership in central sphere of influence;
>
> $|g_R|$, depending on proximity of sender to other parties—which, by postulate [[7]] , as interpreted, translates into sender membership in the group of centrals; and
>
> $|v(j,t'_j)|$, magnitude (in any $\bar{S}$) of receiver space part of velocity—which below will be equated to the rate of internal development of the receiver.

If the substantive interpretations be granted then they would seem to be reasonable mediators of internal power / material capabilities. In the extreme, a change in value of ω by π [radians] will reverse the linkage from positive (affinity for ...) to negative (alienation from ...) the receiver.

Another way of viewing these results is this:

> "Power" is a scalar ("charge") but the *expression* of power in international behavior (in the form of linkages) *is a vector quantity, not a scalar*.

Contrast that expression with the existing political literature, in which both power and its expression are regarded (usually or always implicitly) as scalars. The proposed resolution of the ambiguity of "power potential", i.e. capabilities, versus power expressed behaviorally, is that *the former is a scalar, the latter is a vector*.

Turning to spheres of influence and bimodal polarization and looking at Figure 13.1, the existence of influence spheres seems clear enough if they be equated to the spoke-like pattern of linkages emanating from each central. Reference to equation (17.10) shows the corresponding construct: Relative to a given sender it consists of those receivers for which the value of ω is sufficiently close to zero; being in the sphere of influence of a central means having a (space part of) velocity approximately anti-parallel with the repulsive force felt by the central.

Bimodal polarization, the simplest case, arises when two centrals, C_1 and C_2 are extremely powerful (have large q-values) relative to all other centrals. (Again, this seems, substantively, to be as we should want it.) Then the two strongest repulsive forces will be felt by C_1 and C_2, each primarily due to the other. These forces will be approximately parallel and roughly coincidental with the line connecting C_1 and C_2; and the *forces will be oriented in opposite directions* along this line. [The reasoning is: For a sufficiently great preponderance of central party charge between them, relative to any other centrals, C_1 and C_2 must each provide the primary source of repulsion to the other. Thus, from conservation of momentum, their repulsive force vectors must point in opposite spatial directions (as seen in the $\overline{S}$ frames), each one acting on the other party. These remarks are correct to increasing degree as their central charge preponderance is increasingly great, as the spatial region occupied by centrals is increasingly small, and as their spatial configuration is increasingly static. The latter two conditions act to minimize the complication due to delays in signal transmission between the parties.] If ω_1 is the angle for a given peripheral velocity relative to the force on C_1, then the corresponding angle relative to the force on C_2 is given by $\omega_2 \cong \omega_1 - \pi$ [radians].

Reference to (17.10) then shows that the respective linkages are

$$\theta(j,1,t) \text{ and } \theta(j,2,t) \cong -\theta(j,1,t) \; ; \tag{17.11}$$

that is, the two linkages are negatively correlated. With respect to United States versus Russia, this result is roughly evident in the appearance of a complimentarity in their dichotomous linkages, Figure 13.1, and, especially, in the negative correlation appearing in figure 14.2; thus the preference for equation (14.2) and the data year 1980, in which the Cold War bifurcation is pronounced. In model terms, another way of viewing this is that the repulsive forces on these two parties are aligned approximately with the $\overline{X}_R^1$-axis introduced in Part 15. These data constitute modest and highly qualitative evidence for the applicability of the space-time model. [The diminutive role of Russia as a central, compared with USA, Britain, France, and Japan, in Figure 13.1 is consistent with other dichotomous linkage diagrams based entirely on export data concerning the receiver. The role of Russia as a central is much more noticeable for linkage diagrams calculated from military personnel deployments and arms transfers, both coming from the sender to the receiver, as those terms are used here (Williamson 1985 and 1989).]

We may also observe that if the posited geometry of the social space-time was 1 + 1 dimensional (one dimension each of space and time) then there would be no alternative to polarization; the bimodal world and its result in (17.11) would be automatic. In such a world the constraints α_k , $\alpha_{jk} = 0$ or π would necessarily hold, thereby automatically satisfying the constraints given in equation (12.11). By contrast, in the 3 + 1 model proposed here, bimodal polarization and the Richardson process are possible but not automatic.

Turning, now, to power transition (PTR) theory, one easily matches it to the space-time framework. Let "development" consist of parties moving toward the $\overline{cm}$ (center of mass) of the system. Then, by postulate [[13]], parties in the non-L (i.e. peripheral) group typically are experiencing development (movement toward) or its opposite (movement away from) the $\overline{cm}$. [One could also regard that the interval parameter s , equation (12.1), is providing a different index of internal development, one that accumulates for movements in all directions.] Then the rapid development, item 1 of the PTR discussion in Part 16, corresponds to rapid movement toward the system center, if we let the "challenger" and "status quo" nations correspond to peripheral and central parties, respectively. The possibility of radical change in alignments, item 3, can be seen as the result of the incoming peripheral "striking" the center and disturbing the relative positions of the centrals in the manner of the queue ball breaking the pack in the game of pool—not, as Inis Claude said (1962, p.57), a rear-end collision but a head-on collision. In model terms, this collision happens when the incoming party is in sufficient proximity to exchange local repulsive forces with the centrals. Due to these repulsive forces, the challenger is repelled back into the periphery (out of range of the repulsive local forces) but, upon reaching the top of its trajectory it again falls back toward the center—the recovery or "phoenix" factor (Organski and Kugler 1980), item 4, above, in the present document. That these PTR effects work on preexisting alignments created in prior processes of national development, corresponds to the previous arrival of parties at the system center and the corresponding linkage interactions then formed with the periphery, corresponding to equations (15.5), (15.6), and their applications. Dynamically, the transition process, itself, depends on the momentum imparted to the center via collision which, perforce, must be supplied by the inward falling of a peripheral, thus more recently developed party.

From the viewpoint of PTR theory, a clear limitation in the space-time model is the absence of a global conflict- or war-producing mechanism. We can see the possible outline of such a mechanism: During the process of collision the linkages must be changing rapidly, as the relative positions of the parties—both central and peripheral—are altered by the transfer of momentum to the former. A connection to the possibility of global war may lie in those changes. However I leave that topic for another occasion.

[And returning to Restriction (a), following equation 12.23, note that there is an asymmetry between inward and outward trajectories. Suppose the conventional sense of a Richardson process is a positive reaction to parties in the opposing "sphere of influence"—corresponding to parties situated on the opposite side of the system $\overline{cm}$. Then, for Group L receivers, that convention is satisfied by the inward-moving non-Group L parties (motion toward the $\overline{cm}$; ↔ sender velocities and receiver forces parallel). For outward-moving parties, the reaction coefficient signs are reversed; the change rate is actually inhibited by the non-Group L parties.]

18. Rationale of the Societal Minkowski Space-Time Concept.

Consider again the metric equation (11.1) referred to one of the frames $\bar{S}$. As noted near the end of Part 11, with the aid of equation (8.4) that metric can be written

$$\Delta s^2 = \sum_k (\Delta \bar{x}^k)^2 - (c\Delta t)^2 \ . \tag{11.1'}$$

Define a new parameter

$$\Delta\tau \equiv i\Delta s / c \ . \tag{18.1}$$

This new parameter has the dimensional units of an increment in clock time, in that the latter, according to (8.4), is written as $\Delta t = \Delta\bar{x}^0 / c$; $\Delta\tau$ can be interpreted as change in time as read from a "social clock" carried by the party exhibiting the interval change Δs along its path. (The physical analogue is exactly the elapsed time on a clock carried by a party, possibly moving relative to some given observer.) From (11.1)' and (18.1) one concludes

$$-c^2\Delta\tau^2 = \sum_k (\Delta\bar{x}^k)^2 - c^2\Delta t^2 \text{ and}$$
$$-\Delta\tau^2 = \sum_k (\Delta\bar{x}^k / c)^2 - \Delta t^2 \ , \tag{18.2}$$

which, dividing each term by $\Delta\tau^2$, shows

$$-1 = \sum_k (\Delta\bar{x}^k / \Delta\tau \cdot c)^2 - \Delta t^2 / \Delta\tau^2 \ . \tag{18.3}$$

Now let c become arbitrarily large; then

$$\sum_k (\Delta\bar{x}^k / \Delta\tau \cdot c)^2 \to 0^2 \text{ , as } c \to \infty \ ; \tag{18.4}$$

thus, as $c \to \infty$, (18.3) becomes $-1 = -\Delta t^2 / \Delta\tau^2$ or

$$\Delta\tau \to \Delta t \text{ , as } c \to \infty \ . \tag{18.5}$$

Equation (18.5) shows that, in the limit of infinite societal signal speed, the Minkowski aspect of the space-time disappears; in its place, according to (18.5), there emerges a *single parameter, the increment of ordinary time* (and of ordinary non-space-time dynamic models), describing the progress ("development" in one sense, see above) of each party.

Consider how this issue, concerning the variation of c , affects postulate [[12]] of Table 17.1 . The first of equations (12.3) specializes to $v^i = dx^i / ds$ from which, also using equation (11.2), $v_i = dx_i / ds$. From (11.11), in $\bar{S}$ -type (clock) frames these become $\bar{v}^i = d\bar{x}^i / icdt$ and $\bar{v}_i = d\bar{x}_i / icdt$, from which $(\bar{v}_i\bar{v}^i)^{1/2} = i(d\bar{x}_i d\bar{x}^i)^{1/2} / cdt$. Thus $(\bar{v}_i / \bar{v}^i)^{1/2} \to 0$ as $c \to \infty$, whereby postulate [[12]], that typical $(\bar{v}_i\bar{v}^i)^{1/2}$ be a non-negligible fraction of 1, becomes unsustainable. Take away postulate [[12]] and the emulation of nodal structure and all related effects disappear. In sum, *the social space is a Minkowski space-time so as to accommodate, via velocity—that is, developmental—effects, the various aspects discussed in Part 17*.

19. Conclusions.

What are we to make of the above? Clearly much work still undone is needed, in order to decide the value of the proposed social space-time dynamic model. Whether, and to what extent, nodal structure as described is factually descriptive, at this juncture very little empirical evidence has yet been uncovered; we do not actually know to what historical periods nodal structure applies, other than the 1960s and 1970s; nor do we know that it has evolved in the manner assumed. Evidence for or against the R-process timing delays remains to be developed. Obviously, these matters of factual ignorance should be remedied to the extent that available historical information permits.

In addition, the mathematical and evidentiary reasoning is incomplete. To some extent, the connections given in Part 17 are merely plausibility arguments that the above characteristics lead to various empirical features, or otherwise offer insight. Other than the linkage indicators, we are unclear as to the empirical referents of the variables. As mentioned already, specification of components in the $\bar{S}_k$ and other $\bar{S}$ -class frames is incomplete. The values of social unit "mass" and "charge" remain to be specified.

How might these issues be addressed? In response, an important consideration is that, by emulating, with indicated modification, the electrodynamics of charged particles (postulate [[5]] in Table 17.1), the path seems clear to fully operational computational simulations. For instance, the key question of an appropriate equation to constructively compute the movement of parties over time is well-answered. [See Rohrlich, 1990, pp. 155-156, equations (6-87), (6-88), (6-91) and pp. 188-194. The mathematical series convergence issue raised there is critical.] One approach, then, is to seek to construct and program such a computational dynamic model and to run it (millions of times, perhaps?) using alternative input assumptions, in search of outputs offering a plausible fit to historical observations. The question then would become how to recognize such a plausible fit, to which a satisfactory answer is not yet known. ("Plausible" does not mean exact replication; rather, it means replication of a population of simulate histories of which the empirically observed one is "reasonably"—however construed—a sample.) Additional work, if successful, seeking to make operational the variables, would compliment the simulation approach by supplying reasonable input assumptions. One result, it is hoped, would be a new tool with which to simulate and forecast global political change.

The contribution of this paper takes the different direction of showing why such a space-time approach is already of interest. Aside from raising the goal of computational dynamic forecasting, the answer lies in various new ways of picturing global relations. These are summarized below:

- Common to all the various phenomena discussed above, lies the nodal linkage global network.

- The phenomenon of a global network structure that is nodal, as expressed in linkage terms, is a disguised form of the Richardson process, where the latter is generalized to include variable reaction and grievance coefficients and delayed reaction times, and altered in content to refer to an interaction ("competition") between parties in terms of internal development (rather than armaments).

- The Richardson process is itself constructed from assumptions including rational utility-maximizing choices by the parties but also including a rate of convergence factor (the standard fatigue coefficient). The latter supplies the dynamic (change over time) principle to the model.

- Conversely, the rational choice element is retained in the form of the reaction and grievance quantities. Together with the fatigue coefficient, these quantities find expression in the nodal network pattern: the linkage quantity θ is a function of them [equations (12.9a), the remainder of Part 12 beginning with equation (12.13), and either of (15.4) or (15.6)].

- Abstractly, the nodal structure can be characterized in geometric terms of relative positions, velocities, accelerations, and forces in a particular type of space. The global organization of parties into spheres of influence and, to the extent applicable, in bimodal polarization, receive exact definition in these same geometric terms.

- The distinction (see Part 17.2) between "internal power" ("power potential") and the external behavioral expression of power as influencing behavior, involves the distinction between the *scalar* character of the former and the *vector* character of the latter. Moreover, the "efficiency" of translation from one to the other is expressible, geometrically, as a particular inner product (retarded sender force, receiver velocity).

- The historical evolution of global society, including civilizational convergence and power transition, can be viewed, abstractly, as a problem of geometry and dynamics. In these terms, the history of human society resembles a process of transition from a "hot", amorphous ball of "gas", in the distant past, to a "cool", "condensed", highly structured system in the present; the mechanism of transition emulates the physical radiant cooling mechanism, via acceleration, of classical charged particles.

These features strongly motivate the goal of a computational social space-time dynamic model.

Bibliography and graphics follow, starting next page.

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Figures and Table 13.1.
Follow, starting next page.

List of figures and Table 13.1

Fig. 2.1

The expressions u-curve-1 and ...-2 refer to curves of constant utility for party 1.

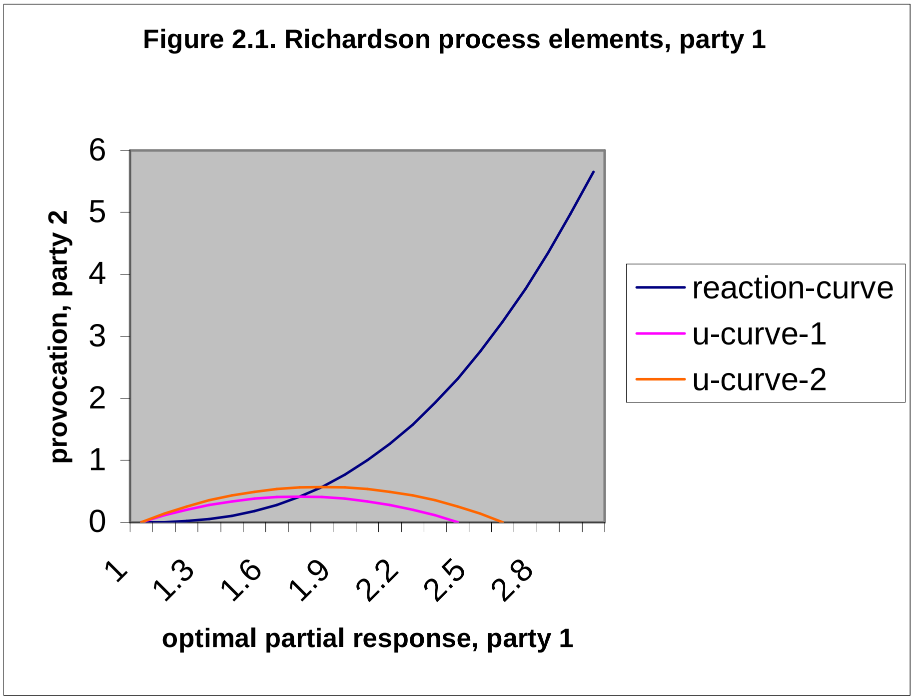


Fig. 4.1

Figure 4.1. Convergence toward partial reaction curve of party 1 in a Richardson process.

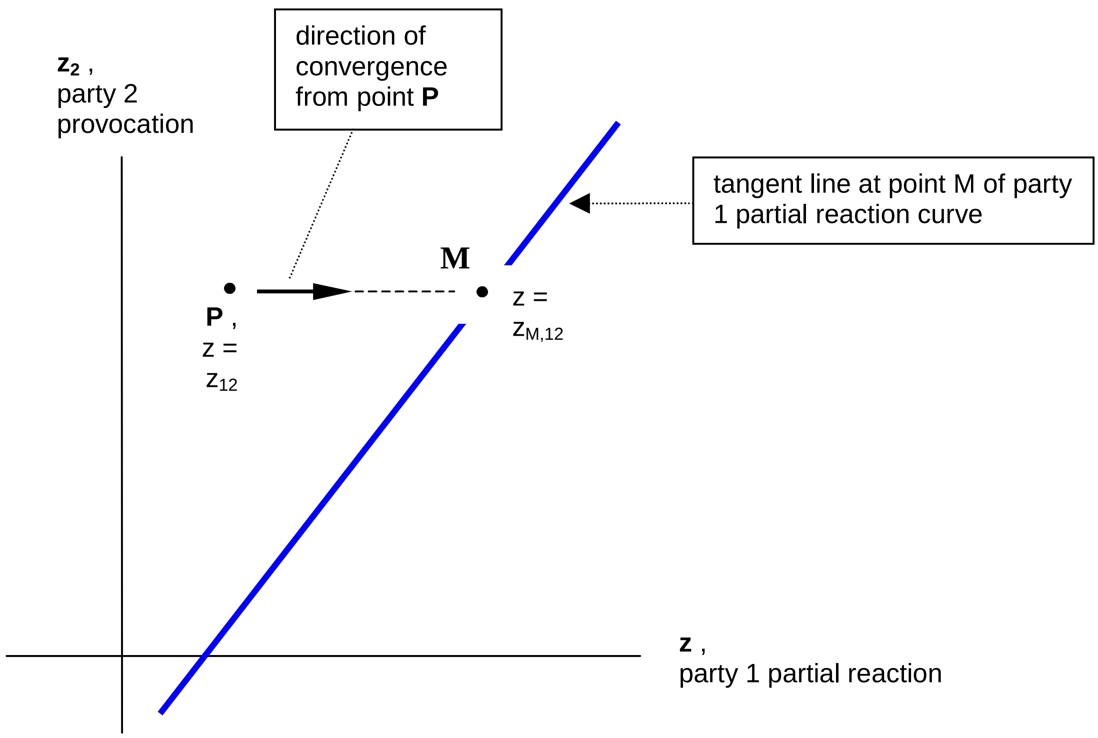


Fig. 6.1

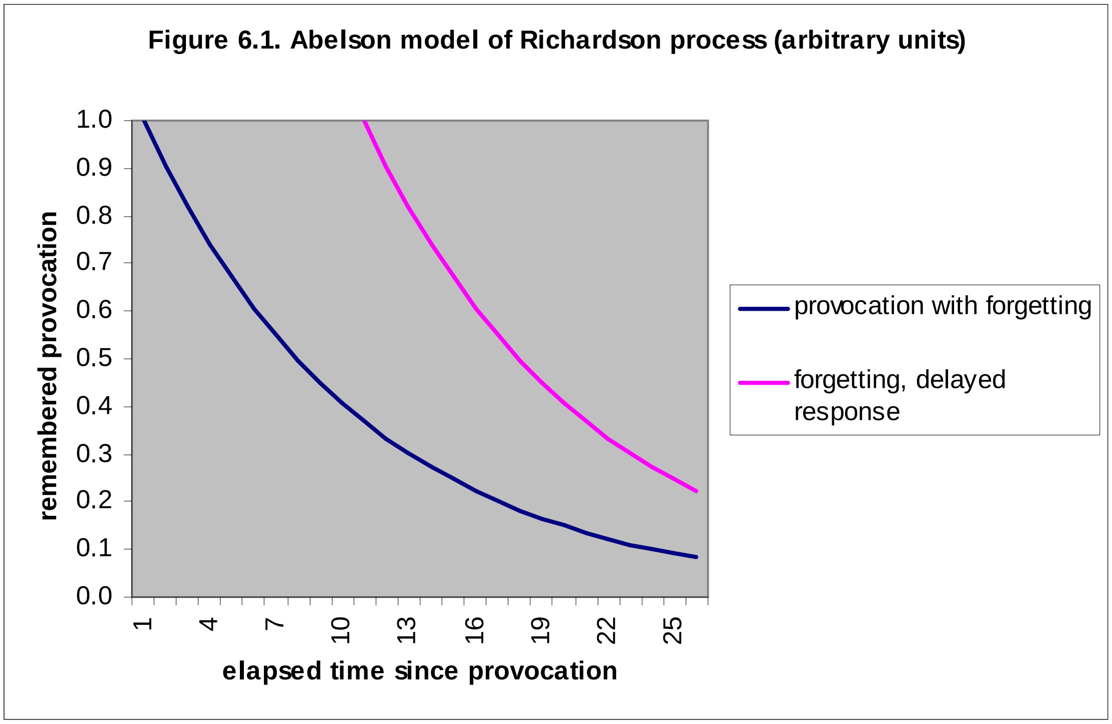


Figure 6.1. Abelson model of Richardson process (arbitrary units)
remembered provocation
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1
4
7
10
13
16
19
22
25
elapsed time since provocation
provocation with forgetting
forgetting, delayed response

Fig. 8.1

Figure 8.1. Social space-time diagram, arbitrary reference frame S

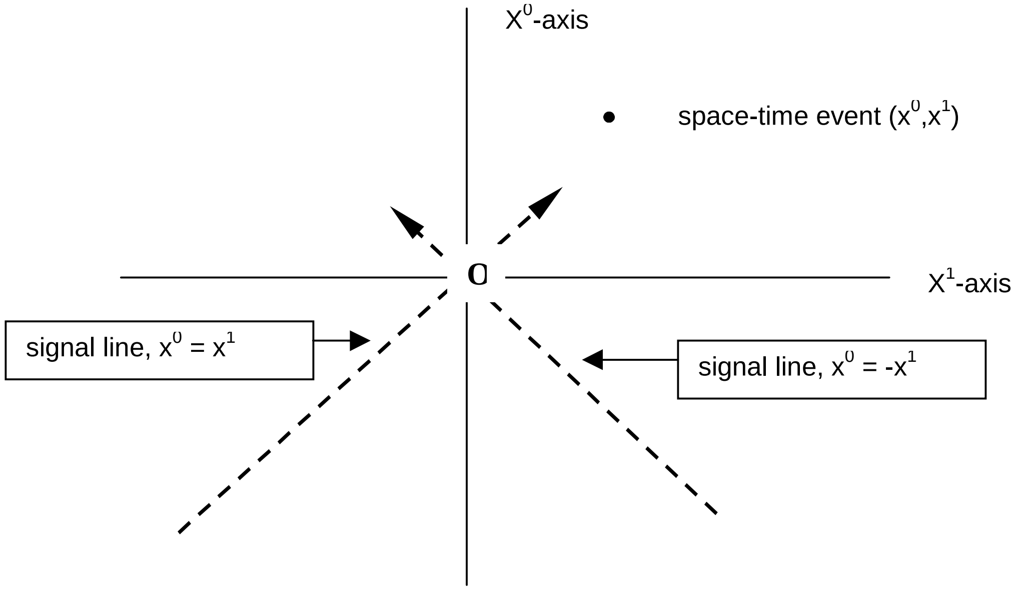


Fig. 9.1

Figure 9.1. Social space-time diagram, clock reference frame $\bar{S}$, showing coordinates $\bar{x}^0 = ct$ (for $\bar{X}^0$-axis) and $\bar{x}^1$ (for $\bar{X}^0$-axis)

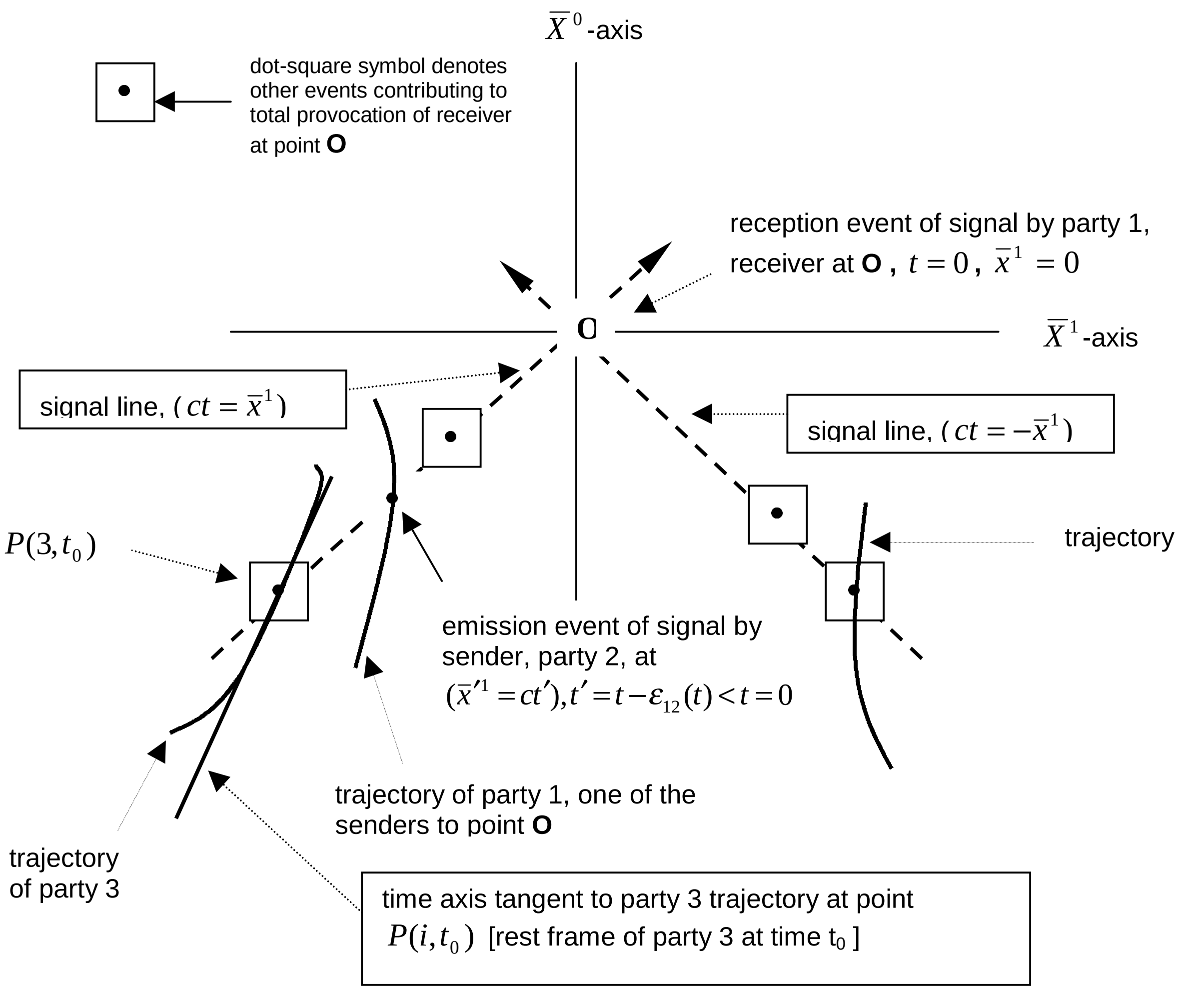
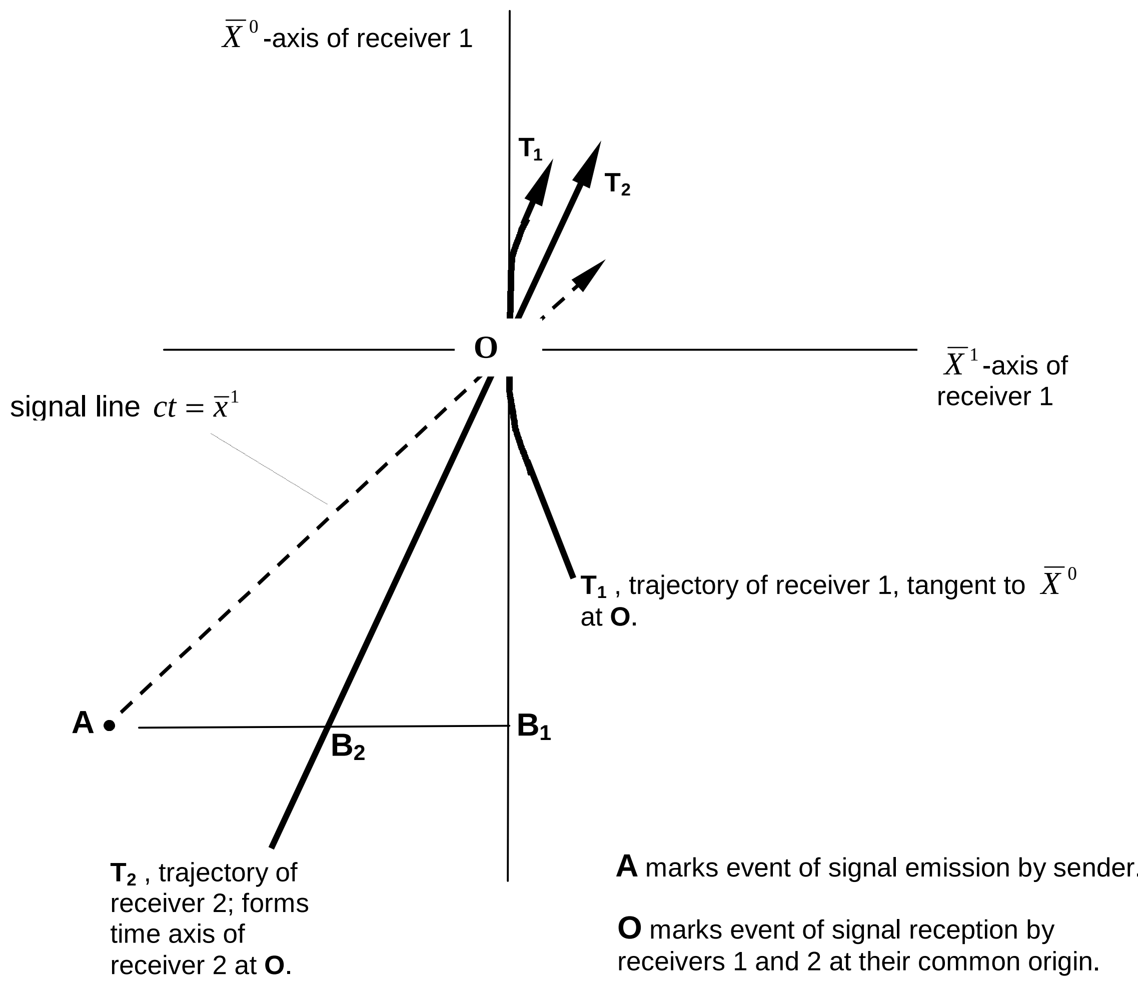


Figure 10.1. Social space-time diagram,
clock reference frame of type $\overline{S}$



Euclidean / Newtonian model (invalid): Line segments $\mathbf{AB_1} \neq \mathbf{AB_2}$ are space displacements corresponding to time displacement $\mathbf{B_1O}$, for signal from **A** to **O** , reference frames with origins at **O** of receiver 1 and receiver 2, respectively.

Fig. 13.1

Figure 13.1. Dichotomous linkages based on directed total trade.

Data year: 1980.
Data source: Barbieri, 1998b .

Numbers in boxes are Correlates of War project nation numbers. See Table 13.1.

Key: receiver → sender

denotes receiver allocated ≥ 20% of total trade to sender. See Part 13, above.

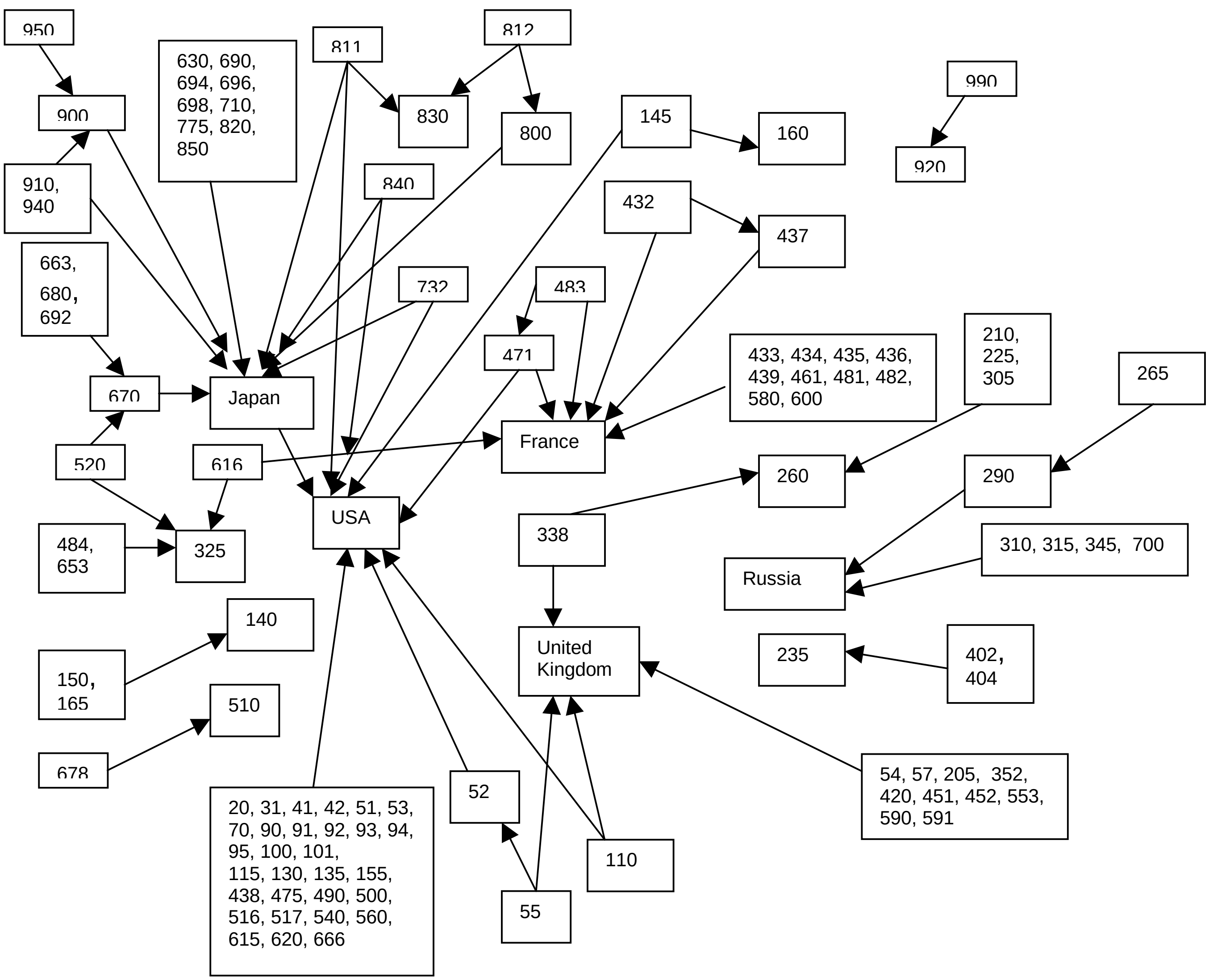


Table 13.1

Table 13.1. Correlates of War project nation numbers appearing in Figure 13.1.

COW number	nation
20	Canada
31	Bahamas
41	Haiti
42	Dominican Republic
51	Jamaica
52	Trinidad
53	Barbados
54	Dominica
55	Grenada
57	St Vincent & Grenadines
70	Mexico
90	Guatemala
91	Honduras
92	El Salvador
93	Nicaragua
94	Costa Rica
95	Panama
100	Colombia
101	Venezuela
110	Guyana
115	Suriname
130	Ecuador
135	Peru
140	Brazil
145	Bolivia
150	Paraguay
155	Chile
160	Argentina
165	Uruguay
205	Ireland
210	Netherlands
225	Switzerland
235	Portugal
260	German Federal Republic
265	German Democratic Rep.
290	Poland
305	Austria
310	Hungary
315	Czechoslovakia
325	Italy/Sardinia
338	Malta
345	Yugoslavia/Serbia
352	Cyprus
402	Cape Verde
404	Guinea-Bissau
420	Gambia
432	Mali
433	Senegal
434	Benin/Dahomey
435	Mauritania
436	Niger
437	Ivory Coast
438	Guinea
439	Burkina Faso (Upper Volta)
451	Sierra Leone
452	Ghana
461	Togo
471	Cameroun
475	Nigeria
481	Gabon
482	Central African Republic
483	Chad
484	Congo
490	Zaire (Congo, Kinshasa)
500	Uganda
510	Tanzania/Tanganyika
516	Burundi
517	Rwanda
520	Somalia
540	Angola
553	Malawi
560	South Africa
580	Malagasy
590	Mauritius
591	Seychelles
600	Morocco
615	Algeria
616	Tunisia
620	Libya
630	Iran (Persia)
652	Syria
663	Jordan
666	Israel
670	Saudi Arabia
678	Yemen Arab Republic
680	Yemen People's Republic
690	Kuwait
692	Bahrein
694	Qatar
696	United Arab Emirates
698	Oman
700	Afghanistan
710	China
732	Korea, Republic of
775	Myanmar (Burma)
800	Thailand
811	Cambodia (Kampuchea)
812	Laos
820	Malaysia
830	Singapore
840	Philippines
850	Indonesia
900	Australia
910	Papua New Guinea
920	New Zealand
940	Solomon Islands
950	Fiji
990	Western Samoa

Fig. 14.1

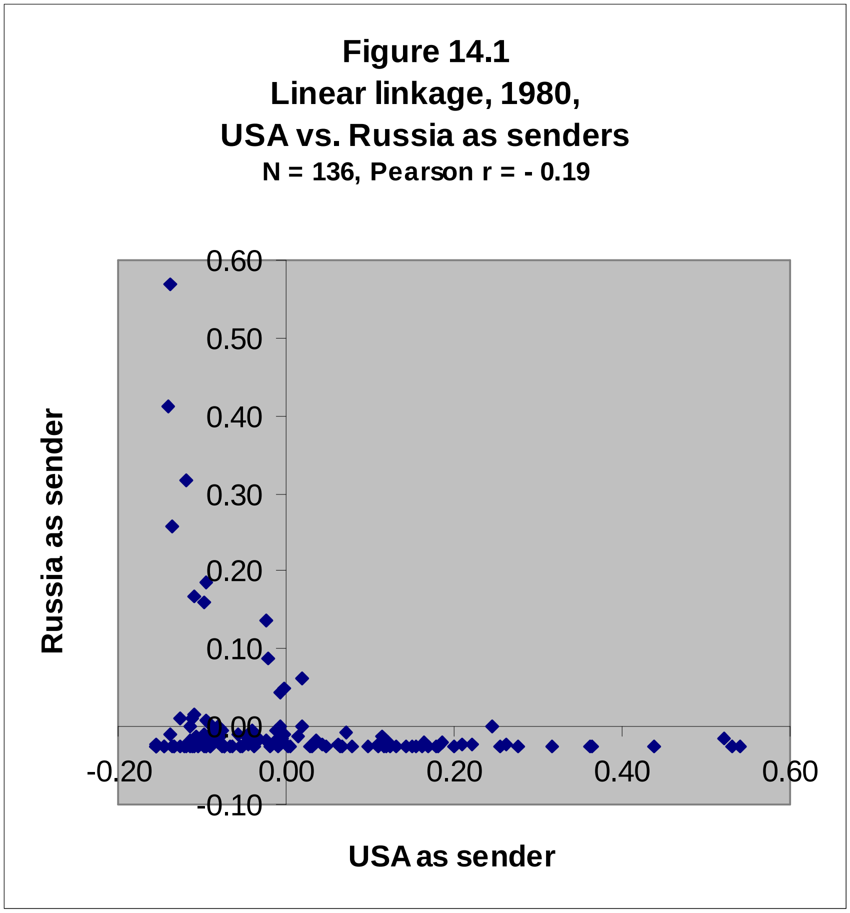


Figure 14.1
Linear linkage, 1980,
USA vs. Russia as senders
N = 136, Pearson r = - 0.19
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
0.00
0.20
0.40
0.60
Russia as sender
USA as sender

Fig. 14.2

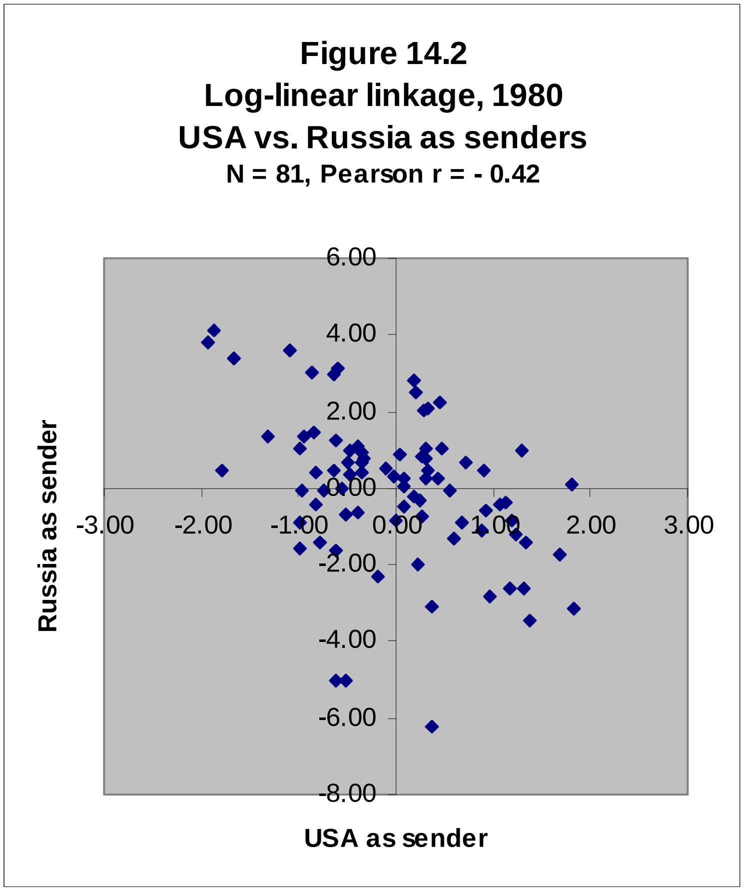


Figure 14.2
Log-linear linkage, 1980
USA vs. Russia as senders
N = 81, Pearson r = - 0.42
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
-8.00
-3.00
-2.00
-1.00
0.00
1.00
2.00
3.00
Russia as sender
USA as sender