Use of an Artificial Neural Network to Enhance an Input-Output Dynamic Model

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Introduction

This paper discusses possible uses of artificial neural networks (Haykin, 1994; Williamson, 1996; Williamson and Bueno de Mesquita, 1997) to enhance input-output dynamic modeling (Leontief, 1986) of the global human-environmental system (Button, Duchin and Kreith, 1997).

An artificial neural network (herein "network") can help input-output approaches to global modeling by 1) representing affects due to "non-economic" factors (political, cultural, environmental, etc.) for which , typically, there is no currently sound theoretical basis for a more explicit treatment, and 2) correcting for errors which arise when a linear model is used to approximate non-linear relationships.

The following discusses two such possible alterations of the basic model. The first uses a network to represent the affects of non-economic factors and of non-linear contributions from economic factors. The alteration consists of a term added to the basic input-output dynamic equation. The second alteration uses a network to represent the evolution over time of the input and capital coefficient matrices (which thus undergo the considerable revision of being viewed as variables rather than constants). Either of these or both at once can be used to modify the basic equation; however the first appears easier to carry out and to have fewer complicating aspects; thus it would reasonably be done first

Before proceeding, perhaps we should also note that input-output modeling is not usually presented in a way that emphasizes or makes clear its dynamic element, that is, the principle by which the system variables are posited to change over small increment dt in time, or (in finite form) from one period to the next. This element must be at least implicit, however, in applications where the model is used to forecast how the system will be affected at some future time, as a function of the state of the system at referent (the "present") time. What I do below is to start with this "implicit" form, as given in the indicated source, and work for there to recast in a different form in which it is easier to make the indicated generalizations of the model.

Basic Input-Output Equation Rewritten

To proceed, it is helpful to rewrite the basic dynamic equation in an equivalent form. Let t denote a reference year of known values, from which is to be predicted future values in the year t + 1. An input-output dynamic model is then given (Leontief, 1986, p. 31, eq. 2-14) by

$$x(t) - Ax(t) - B[x(t+1) - x(t)] = y(t) \qquad (1)$$

where

x = a column matrix of production sector output levels
y = a column matrix of deliveries to final users
A = input coefficient matrix
B = capital coefficient matrix.

Equivalently, eq. (1) can be rewritten as

$$Bx(t+1) - Bx(t) = [I - A]x(t) - y(t)$$

or

$$\Delta x(t) = \Delta \bar{x}(t) \qquad (2)$$

where

$I =$ identity matrix

$\Delta x(t) \equiv x(t+1) - x(t)$

$\Delta \bar{x}(t) \equiv B^{-1}[I - A]x(t) - B^{-1}y(t)$ (3).

The more realistic version of eq. (2) includes an error term:

$$\Delta x(t) = \Delta \bar{x}(t) + \varepsilon\,, \qquad (4)$$

which I will use as the basis of what follows below.

If there are many distinct behavioral units to be represented (e.g. nations, geographic regions, or other groupings), this equation and its modifications, below, might need to be defined separately for each; and additional terms might be required to represent the trade and other linkages among them. For purposes of illustration I am ignoring such complications here.

Using a Network to Represent Non-economic and / or Nonlinear Factors

Let $z^i(t), i = 1, \ldots p$ denote values of p-many global systemic variables (possibly including x and y in non-linear combinations) for t and the years preceding t. Let $\xi[z^i(t)]$ denote some (generally) non-linear function of the $z^i(t)$, where ξ is a column vector defined as the outputs of a single neural network (or a set of networks--both schemes are possible). Then the *first altered* input-output dynamic model is

$$\Delta x(t) = \Delta \bar{x}(t) + \xi[z^i(t)] + \varepsilon' \quad . \qquad (4')$$

In neural network modeling, the “dependent” value to be estimated by the network is called the *pattern* variable. In this proposed modeling approach, the pattern variable of the function $\xi[z^i(t)]$ would be $\Delta x(t) - \Delta \bar{x}(t)$.

Possible sources of the $z^i(t)$ include the very considerable data archives of various sort concerning economic indicators, cultural indicators, public opinion surveys, systematic international and comparative data sets (such as maintained by the Inter-university Consortium for Political and Social Research, Ann Arbor, MI), etc.

Using a Network To Represent the Evolution of Input and Capital Coefficients

This option is more problematical because it presumes the possibility of empirically estimating the A and B matrices at many distinct time points (say, at quarterly intervals since 1945).

Supposing this were possible, let

A(t) = input coefficient matrix
B(t) = capital coefficient matrix

both evaluated at time t .

Let $\alpha[z^i(t)]$ and $\beta[z^i(t)]$ denote two additional functions of the $z^i(t)$, each of them a matrix defined as the outputs of its own neural network (or set of networks). Then the pattern variables for neural network evaluation of these new functions would be A(t) and B(t), respectively.

The *second altered* input-output dynamic model, combining non-economic, nonlinear, and variable coefficient alternations, is then

$$\Delta x(t) = \Delta\chi(t) + \xi[z^i(t)] + \varepsilon'' \qquad (4'')$$

where

$$\Delta\chi(t) \equiv \beta[z^i(t)]^{-1}\{I - \alpha[z^i(t)]\}x(t) - \beta[z^i(t)]^{-1}y(t) \qquad (3')$$

The idea is that the models represented by equations (4), (4′), and (4′′) constitute successive improvements because they take increasingly comprehensive account of the relevant factors. Hopefully, the corresponding error terms ε , ε' , and ε'' for otherwise identical tests (same variables and cases) would show declining variance; however this is not assured in non-linear modeling and would be a matter to be found empirically.

References

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