**New Considerations About Military Personnel Dynamics;
Evidence Against the Standard Richardson Arms Race Model.**

Paul Williamson, Ph.D.

Research Report 0201
Global Vision, Inc., P.O. Box 4394, Ann Arbor, MI 48106-4394
paulrw@globechange.org

July 2002

1. Background.

This report is the second, of research done in support of the global modeling development program of Global Vision, Inc. For background on this program and on prior work supporting it, the interested reader may wish to consult the first report (Williamson, 1996), an article (Williamson and Bueno de Mesquita, 2000), and the Global Vision, Inc. web site at www.globechange.org . The report is available as an e-mail attachment or paper print, upon request to paulrw@globechange.org . The article is available on the World Wide Web at http://www.crp.cornell.edu/peps/Journal/Vol6-No4/Rsr-Williamson_deMesquita.pdf .

In brief, this program has so far focused on exploring the use of artificial neural networks to forecast changes in social indicators that are influenced by unknown or highly complex processes. As a point of departure, the work has been further focused on predicting changes in numbers of military personnel in national establishments and, initially, just to the military personnel of the United States. Since the first report was written in 1996, the scope has been enlarged; first to include the military personnel of 6 other major nations—China, France, Germany, Japan, Russia, and United Kingdom—in addition to the United States; then to include the military personnel of all nations comprising the COW-defined international system as defined (Correlates of War project, http://www.umich.edu/~cowproj/ ; Peace Science Society, International http://pss.la.psu.edu/). The basis of selecting these seven nations is discussed in Appendix 1.

In this report I ignore most of these more recent studies except to mention that, to date, they have not been as successful as the USA-alone case, in finding a promising neural network based predictive model for military personnel changes. (The work to examine various candidate networks continues, to which is now added the task of forecasting economic indicator changes, with emphasis on the impact of deadly conflict on economic growth.) Instead, here I wish to call attention to an interesting result that occurs without the aid of the neural networks so far tested; indeed, is equally strong without them.

2. Second-Order Changes in Logarithm of Military Personnel.

One feature of the study is that the changes examined were in the natural logarithms of numbers of personnel, not in the personnel numbers themselves. In common with other measures obtained by counting the number of some class of items, reporting a negative number of military personnel would not be a meaningful observation; in other words, numbers of military personnel are bounded from below by zero. When presented by a variable that is so bounded, one can always take the logarithm of any of the numbers that are not zero. In the case of national military establishments, most have a non-zero number in most years because they actually have people in the establishments; thus the logarithms are defined and can be taken. (Some nations in normal times, such as Costa Rica in most years, plus militarily occupied nations such as Germany and Japan in the years immediately after 1945, are exceptions.)

One characteristic of logarithms is that a proportionate change—that is, the same fractional change—in value, of the argument (the number from which the logarithm was taken), has the same logarithmic increment regardless how large or small the base number. (For instance the 1/10th increase from a base of 10,000 to 11,000 military persons has the same logarithmic value as the 1/10th increase from 1,000,000 to 1,100,000.) The study reported here used such logarithmic evaluation. This feature of same values for same proportions of change is significant in helping us see the result reported in the next section. It is also significant to interpreting the result, as we will see in the mathematical discussion section.

A second feature of the study is that it was based on second-order changes, defined as follows. Let $y(t)$ be the logarithm of the number of military personnel for a given nation in the given referent year t . Define $\Delta y(t) \equiv y(t) - y(t-1)$; that is, $\Delta y(t)$ is the change in $y(t)$ compared with the previous year value. We can call this the first-order change. Now define $\Delta^2 y(t) \equiv \Delta y(t) - \Delta y(t-1)$; that is, $\Delta^2 y(t)$ is the change in the first order change of the referent year, compared with the first order change of the preceding year. Call this "change in the change" the second-order change.

3. Procedures and empirical results.

The first procedure followed was to scatter-plot the data values of $\Delta^2 y(t+1)$, variously called the "pattern", "output", or "dependent" variable, versus the corresponding values of $\Delta y(t)$, called the "input" or "independent" variable. That is, the second-order changes in the year *following* the referent year were plotted versus the $\Delta y(t)$ first-order change values of the referent year. (The lagging of input year t behind output year $t+1$ reflects the interest in creating a dynamic model, by which is meant a means of estimating some future condition based on knowledge of present conditions.)

This procedure was done for all nations in all years that were in the data set, with results shown below. This data set consisted of all nation years for which data were available on each of several other candidate input variables, of which there were 73. The impact of these 73 other variables was to constrain the number and selection of cases used in the present study. The purpose of the constraint was to make results comparable to results from other studies in the program that did use these other variables. (These other studies are not yet finished. Such partial results as I do have, so far, appear to be of inferior predictive accuracy to those reported here.) With that qualification on case selection, all available nation-years of data from the COW project were used, which provided a total of 5940 cases.

We can see the result in Figure 1. The value of the Pearson product moment correlation statistic $r = -0.73$ confirms the visual impression: There is a tendency for a strong positive rate of increase in military personnel in one year to be followed by a strong decrease in (a "break" on) that same rate of increase the following year; conversely, a strong decrease in military personnel tends to be followed the next year by a strong increase (a "restoration") in the rate of increase. Returning to a point in the previous section, this relationship is in the change in personnel relative to the previous number of personnel, so we see it *because* we are using logarithms of the numbers.

Text continues page 4.

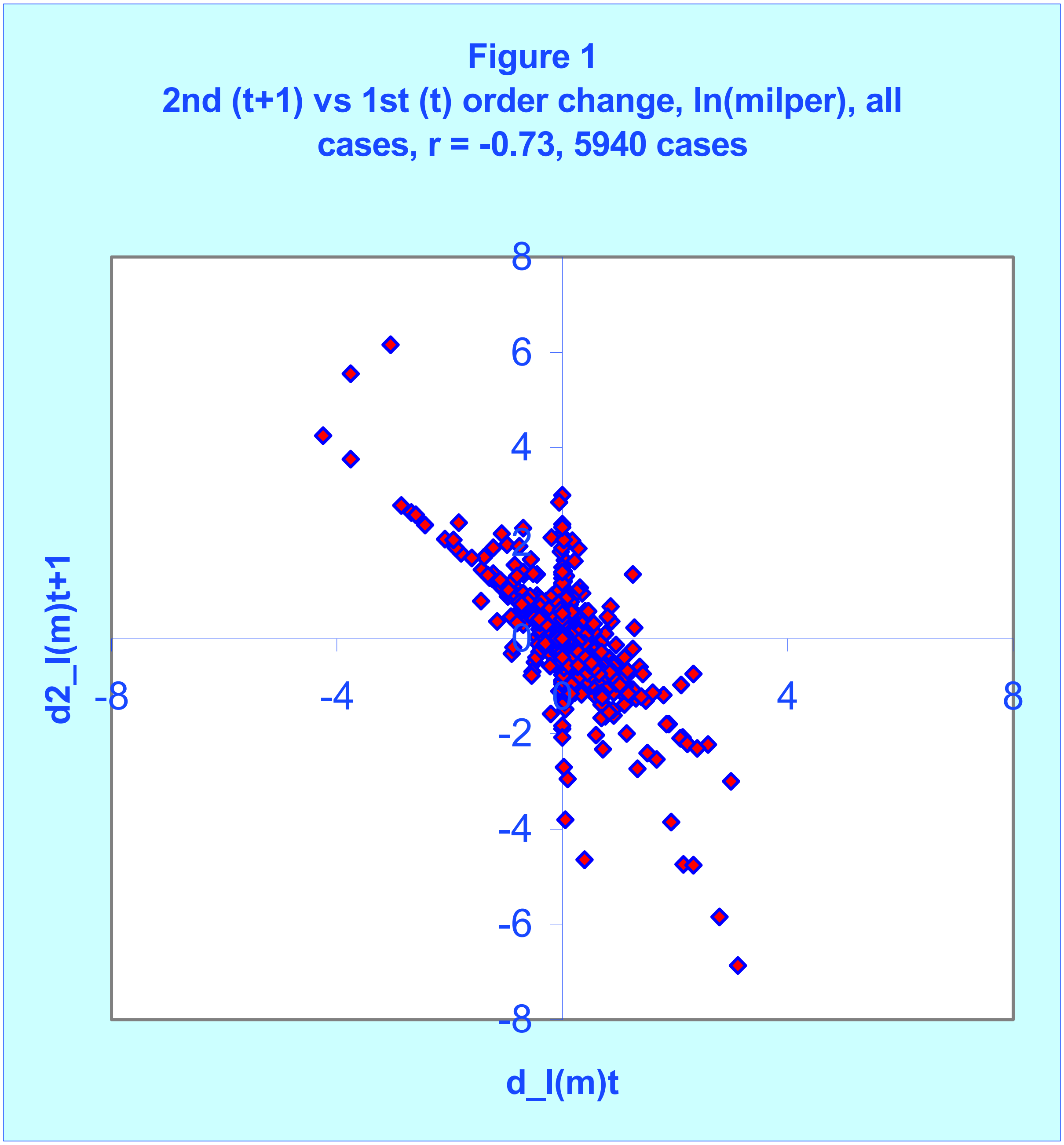
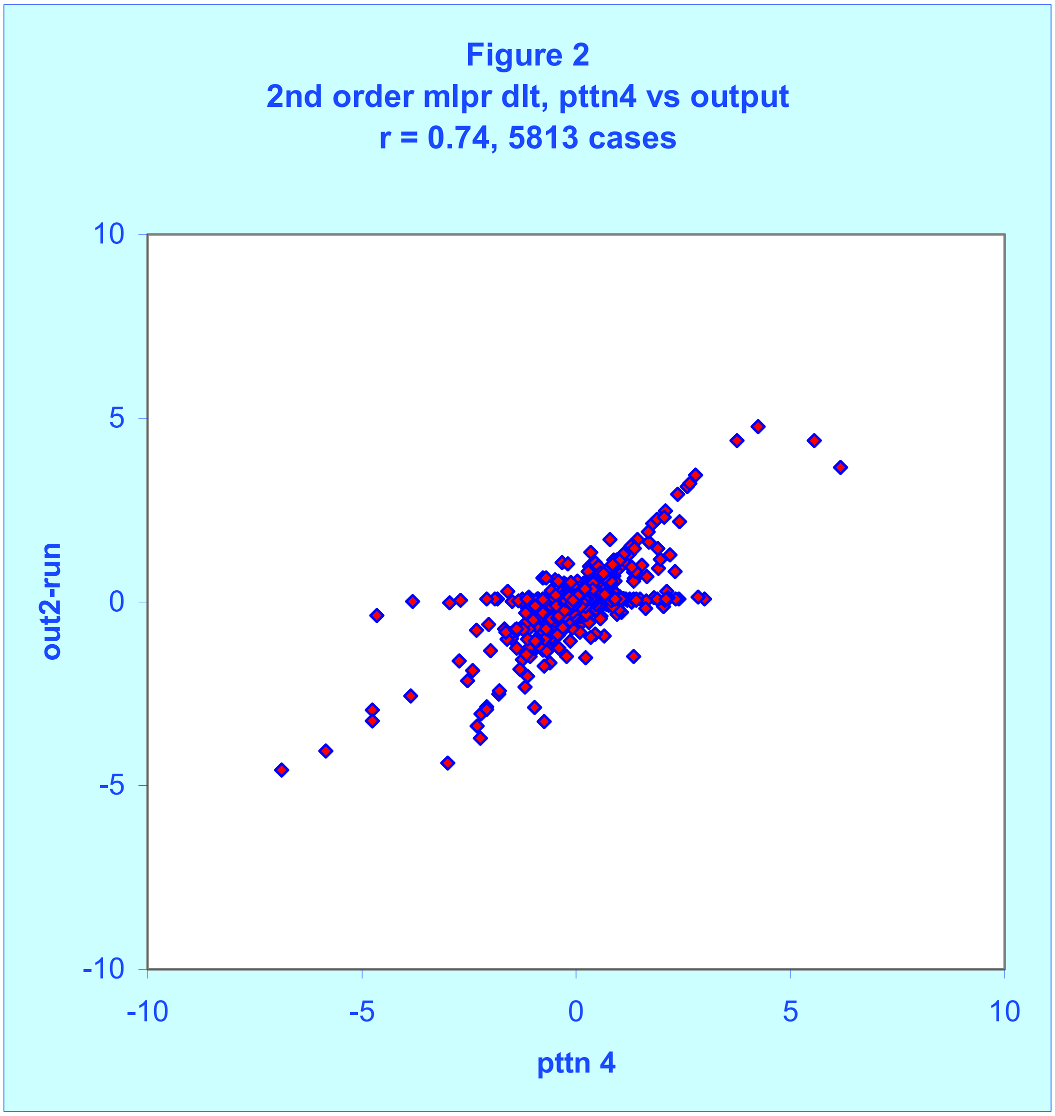
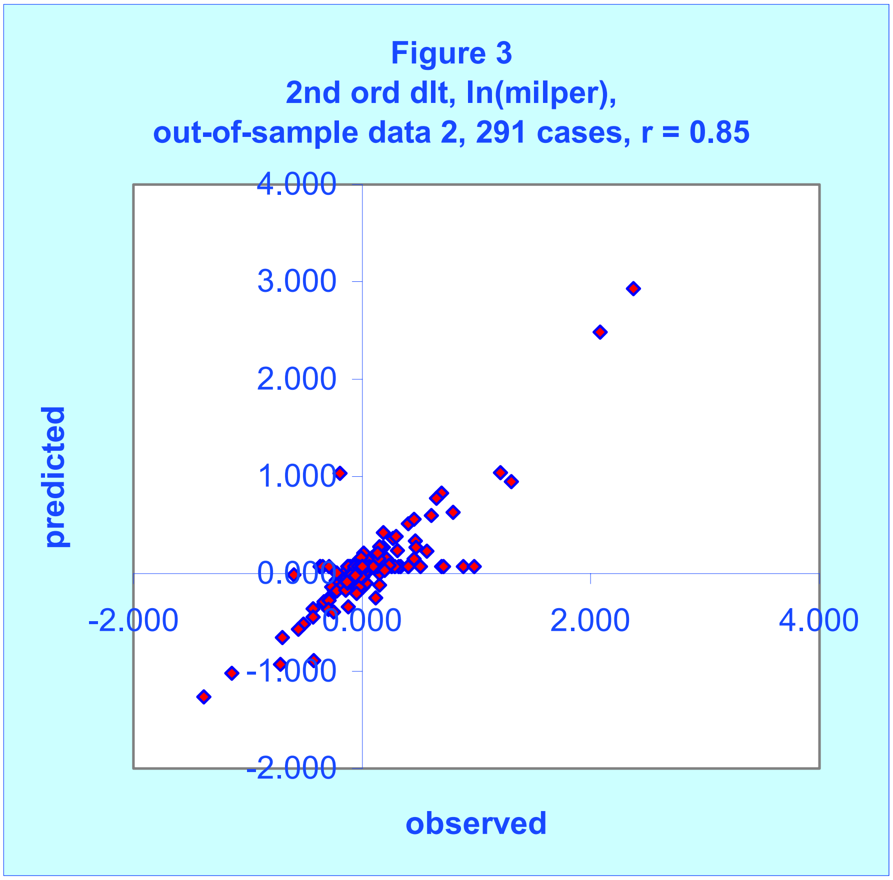


Figure 1
2nd (t+1) vs 1st (t) order change, ln(milper), all cases, r = -0.73, 5940 cases
8
6
4
-2
-4
-6
-8
-8
-4
0
4
8
d2_l(m)t+1
d_l(m)t



In an attempt to find a stronger relationship a second procedure, using artificial neural network modeling, was followed. Again, $\Delta^2 y(t+1)$ was the output variable and $\Delta y(t)$ was the input. Other procedures were followed in which some or all of the other 73 inputs were used, in addition to $\Delta y(t)$. Surprisingly, the best result of these appears to be shown by the neural network in which *only* $\Delta y(t)$ was input. At that, the improvement was only one percentage point, to $r = 0.74$. This result is shown in Figure 2.



Note that the change in sign of the relationship, between figures 1 and 2, reflects that the network training procedure was programmed to optimize the fit to the positive value of $\Delta^2 y(t+1)$; the positive sign could have been arranged for the correlation in Figure 1, also, by substituting the negative of the input in its place. This neural network used 10 intermediate or "hidden layer" nodes. In this study 5813 cases were plotted; this differs from the first procedure by the omission of 5940 – 5813 = 127 cases, corresponding to data for years 1990 and later. These were withheld for use in an out-of-sample test that has not yet been performed. (The idea was to answer previous criticism, by testing cases that followed all the cases used in calibrating or "training" the neural network. This test will happen once a good enough performing candidate network has been selected for it.)

In addition, the remaining 5813 cases were separated into a calibration (training) set of 5522 cases, used for actually developing the network, and a second out-of-sample test set consisting of the remaining 291 cases. Unlike the 127 case out-of-sample test set mentioned in the previous paragraph, this set of cases was randomly selected from the 5813 cases. For the best performing network, this random test set showed a correlation of $r = 0.85$; its scatter plot is shown as Figure 3. It is important to note that performance of the random out-of-sample test set was actually the criterion for choosing the one network out of the many tested, of which we see the results here; so these findings do not have inferential statistical validity. (I am not bothered by the fact that this runs counter to prevailing social science practice emphasizing the role of inferential statistics; in my view the appropriate occasion for such statistics can come after we have found one or more forecasting models that meet many other criteria of comprehensiveness and descriptive accuracy. We are still a long way from that.)

I omit showing plots of results obtained by neural networks using other inputs in addition to $\Delta y(t)$. The corresponding r-values for the 291 random out-of sample cases, plus a brief identification of the additional inputs, are:

1) $r = 0.82$. Added inputs: 5-year, previous to referent year, moving changes in the information-theoretic entropy of energy consumption, calculated on the basis of 7 major powers (China, France, Germany, Japan, Russia, and United Kingdom, United States; plus the same except calculated for military expenditures. For the curious, I show these data in Appendix 1, below.

2) $r = 0.74$. Added inputs: 1st through 4th year lagged values of $\Delta y(t)$.

3) $r = 0.72$. Added inputs: those of item 2) above; plus referent year and 1st through 4th year lagged values of changes in logarithm of military expenditure; plus 5-year previous logarithm of military personnel and the same for expenditure; plus referent year logarithm of energy consumption and change in same from prior year, referent year logarithm of total population, and major-minor political power status for the referent nation.

4) $r = 0.57$. Added inputs: 73 inputs, including all those named above. (See Appendix 2, below.)

5) $r = 0.04$. Added inputs: Same as for item 1) above, but a second neural net was developed, using as pattern the residuals obtained after subtracting the best estimate using the first network based on $\Delta y(t)$, alone.

Again, these results are inconclusive. For greater conclusiveness, they would need to be supplemented, both by studies using additional input combinations and by checking forecast accuracy on all cases and on the post-1989 sample, in addition to the random out-of-sample case results reported above. With those qualifications, the main points are a) none of the factors are shown to have improved network accuracy; indeed, b) the more factors are added, the worse the network performed. This might come about in the following way. Adding more inputs increases the dimensionality of the configuration space of the network. The degraded accuracy is consistent with the premise that these additional dimensions allow many additional local error minima to appear in the configuration space, most of which are very shallow. Then, in the training or calibration process, the network most likely picks one of these shallow minima toward which it then evolves. If that is truly the nature of the problem, an approach like that followed in case 5, above, would be a possible way around it, since then the other inputs are allowed only to influence a secondary network based on the residual outputs after use of the main input $\Delta y(t)$ to define a primary network. If so, the inputs clearly will need to be other than those used in case 5.

4. Mathematical discussion.

Except for the appendices, from this juncture forward I restrict attention the initial correlation finding illustrated in Figure 1. The correlation statistic $r = -0.73$ corresponds to just over half, 53%, of the variance in the dependent variable. In what follows, I additionally disregard the residual 47% of variance. With that qualification, the data are consistent with a linear relation between $\Delta^2 y(t)$ and $\Delta y(t)$. In the following, imagine that there is an underlying continuously varying quantity, the logarithm of military personnel. Call this variable $x(t)$. Then we may define the rate of change of this variable with time, $v(t) \equiv dx(t)/dt$, of which $\Delta y(t)$ for varying times

t are empirically observed samples. Correspondingly, we may imagine that the $\Delta^2 y(t)$ are empirically observed values approximating the second derivative $dv(t)/dt = d^2 x(t)/dt^2$.

Then the plots in Figure 1 support the idea that

$$\frac{dv(t)}{dt} = -av(t) \text{ , for a constant } a > 0 \text{ .} \tag{1}$$

We may also imagine that a is approximated by the slope of ordinary least squares regression of $\Delta^2 y(t)$ on $\Delta y(t)$. Recalling the discussion in section 2, above, equation (1) is approximately accurate because $x(t)$ corresponds to changes in the logarithms of numbers of personnel rather than in the numbers, themselves.

A solution of equation (1) is

$$v = ke^{-at} + c \tag{2}$$

where k and c are constants. [Equation (2) is a solution to (1) because $dv/dt = -ak\exp(-at) + 0 = -av$.] However, we know from Figure 1, also from Equation (1) that $v \cong 0$ when $dv/dt = 0$; that is, when $v(t)$ is constant it is approximately zero. Thus, from (2), $c \cong 0$, which one may take to be exact; thus amend (2) to read

$$v = ke^{-at} \tag{3}$$

Equation (3) implies

$$v(t) \to 0 \text{ as } t \to \infty \text{ .} \tag{4}$$

In words, the logarithmic change rate of military personnel approaches some constant limit c , which one may take to be zero, as time increases. Looking at Figure 1 as well as equation (3), we see that this limit is approached both from above (military personnel increasing) and from below (military personnel decreasing); in the former case $k > 0$; in the latter, $k < 0$.

How are we to interpret, realistically, these equations? In effect, they say that the underlying tendency is for number of military personnel, as represented by $x(t)$, to be constant and, when it is not, equation (1), equivalently (3), governs its restoration to its norm. We may imagine that these equations hold almost all the time but, occasionally, some random external (to these equations) disturbance, for instance a war, war termination, or economic dislocation, suddenly deviates $v(t) \equiv dx(t)/dt$ above or below zero. After the disturbance has passed, the change rate $v(t)$ converges back to zero.

Note that such a disturbance is conceived above as a shift in change rate $v(t)$, *not* in $x(t)$. This implies that $x(t)$ will itself be changed to a new normal value, every time there is a disturbance. Once $v(t)$ has again gone to zero (that is, no longer differs appreciably from it) this new normal logarithm of military personnel value will be wherever $x(t)$ now finds itself. Quantitatively we can write this as

$$\begin{aligned} x(t) &= x(t_0) + \int_0^{\Delta t} v(\tau) d\tau \\ &= x(t_0) + \int_0^{\Delta t} k e^{-a\tau} d\tau \\ &= x(t_0) + \frac{k}{a}[1 - e^{-a\Delta t}] \end{aligned} \tag{5}$$

where t_0 is the last moment prior to the sudden disturbance, $t > t_0$ is the present moment subsequent to the disturbance, $\Delta t \equiv t - t_0$, and τ is a dummy variable of integration. To imagine that the normal value of $x(t)$ occasionally shifts to any new normal value (as at war termination, foreign military occupation, or economic dislocation), the above require that we conceive of this change process as a shock to $v(t)$, after which the same equations govern the shift to the new norm.

From the final expression on the right of (5), we can see that the new equilibrium value $x(t)$ is larger than the old value $x(t_0)$ if $k > 0$, and smaller if $k < 0$. Also, observe that nothing prevents the constant k from differing between one nation and another; and the above reasoning calls for it to be viewed as varying within the same nation from the occasion of one disturbance to another. Differing values would simply mean that specific nations on specific occasions started from different abnormal positions above and below constant military personnel. In addition, the normal (convergent, equilibrium) value of $x(t)$ can differ among nations (as, of course, it sensibly must) as well as over time.

All these speculations are consistent with (but of course not proven by) the data displayed in Figure 1. On the other hand, these data strongly support the generalization that the exponential coefficient a defining the rate of convergence in equations (1) through (3) and (5) *must* have approximately the same value for all nations in all the years examined.

5. Comparison with Lewis Richardson's arms race equations.

Lewis Richardson's (1960) equations can be expressed as

$$\begin{aligned} dw_1 / dt &= \alpha w_2 - \beta w_1 + g \\ dw_2 / dt &= \gamma w_1 - \delta w_2 + h \end{aligned} , \tag{6}$$

where w_1 and w_2 are indicators of military effort by two competing nations; g and h are constants; and $\alpha, \beta, \gamma, \delta > 0$, and are all constant. If one takes the logarithm of numbers of military personnel of a nation as furnishing one reasonable indicator of military effort, then one can draw some comparisons with the model set forth above in equations (1) through (5). First, the arms races equations (6) are explicitly interactive; each party's response depends on the arms level of the other party, whereas the most nearly similar appearing analog above, equation (1), is not. Focusing, now, on the first equation of (6), suppose that $\alpha, g = 0$. The result is

$$\frac{dw_1}{dt} = -\beta w_1 \, , \tag{7}$$

which shows equation (1) resembles a Richardson model in which there is no response to the putatively rival nation and no aggravating or ameliorating factor g. But notice that, if the $x(t)$ in the above is the military effort variable, then $dv(t)/dt = d^2x(t)/dt^2$ in (1) is its *second* derivative, whereas the dw_1/dt in the Richardson result (7) is the *first* derivative of military effort. Of course one could say that, precisely because of the approximate accuracy of equation (1), and because $v(t) \equiv dx(t)/dt$ has the same appearance as w_1 in (7), $v(t)$ is clearly the more appropriate indicator of military effort. If so, then equation (7) cannot be accurate in its original meaning--that is, if we identify w_1 with $x(t)$--because from it we deduce that

$$\frac{d^2 w_1}{dt^2} = -\beta$$

a constant, in contradiction of equation (1).

My purpose in making these comparisons is to show that the empirical finding expressed as Figure 1 appears, in the several ways just indicated, to call into question the Richardson equations as they are normally understood. Probably most significant is the *apparent* independence of military personnel changes from an interactive term (on this point see also Williamson and Bueno de Mesquita, 2000). On the other hand, the approximately 50% of variance in Figure 1 that remains unaccounted leaves plenty of room for more to be added to the story, which might include interactions between nations. Moreover, this finding concerns personnel, not the perhaps more customary expenditure, as the indicator of military effort.

Appendix 1. Negative Entropy Calculations Mentioned in Part 3, Numbered Items 1, 4 and 5.

Again, the results of greatest interest do *not* involve the entropy data, which are described here for the curious. Entropy calculations were done using data from seven nations: China, France, Germany, Japan, Russia, United Kingdom, and United States. The basis of selecting these 7 nations was 1) all were classed as major powers at some time during the study period of 1816 through the most recent data year in the early 1990's; 2) the named state or its plausible successor needed to be defined for the entire study period (resulting in the omission of Austria-Hungary); 3) data needed to be available for all years or for sufficiently many years to allow reasonable estimation based on log-linear interpolation or extrapolation (resulting in the omission of Italy). Estimation was applied to one or more data years of all nations except France and United States (all others having missing data in one or more places). As further discussed below, for the post-1945 periods of military occupation by foreign states, the contributions of Germany / West Germany and Japan, to the military expenditure calculations, were defined as zero.

The basis of calculation was the information-theoretic expression for "relative negative entropy" N given as

$$N = (H_{\max} - H)/H_{\max} \tag{8}$$

where $H_{\max}$ is the maximum possible value of H which latter, in turn, is the information theoretic entropy (Khinchin 1957). The latter is given by

$$H = -\sum_{j=1}^{p} w_j \ln w_j \tag{9}$$

where $w_j = c_j / \sum_{k=1}^{p} c_j$ and c_j is the value of an attribute of party j. Thus, in this context, w_j refers to the fraction of a given capability held by a given nation, relative to the total held by all members of the group. In this case, the group consisted of the 7 nations just mentioned and the capability types were energy consumption and military expenditure, the negative entropy measure being calculated separately for each type; so $p = 7$ in (8) and (9). For years immediately after 1945 when one or both of German and Japanese military expenditures were zero, the corresponding terms of equation (9) were defined as $w_j \ln w_j = 0$. The maximum entropy value occurs in the event that $c_j =$ constant for all parties j. In that case $w_j = 1/p$, so that $H_{\max} = \ln p = \ln 7$ in this instance. From (8), N has maximum and minimum values of 1.0 and -1.0, respectively.

These formulas were used to compute negative entropy scores N for each year in the study period. The values thereby obtained were used to calculate the *change* in negative entropy scores over 5-year and 10-year periods. These 5- and 10-year entropy change values based on energy consumption and, separately, military expenditures, are shown in Figures 4 and 5, respectively. The suffixes "…_n-5" and "…n-10" denote 5- and 10-year values, respectively.

Appendix 1 figures appear next page.

Appendix 2. List of Inputs to Neural Network Studies.

As in the case of the entropy discussion, the following list is provided as a matter of interest concerning forecasting inputs that did *not* work well in the models described in this report. Note, however, that neural networks, being nonlinear, are tricky; some different combination drawn from the same set of inputs might work very well, just as first order change in log military personnel did, when presented by itself.

Inputs numbered 1 through 50 are directly descriptive of the referent nation (that is, the nation being described by the particular case. Inputs numbered 51 through 71 actually describe the named major power; these inputs, year by year, are repeated for each referent nation. The idea was that what was happening among major powers might disproportionately affect what was happening with all other nations (as well as with the major powers). Similarly, the idea of the entropy inputs, 71 and 72, was that the system might be affected by capability distributions among the majors. The results of the particular models tested do not support these ideas.

Appendix 2 table starts page 13; citations appear page 14.

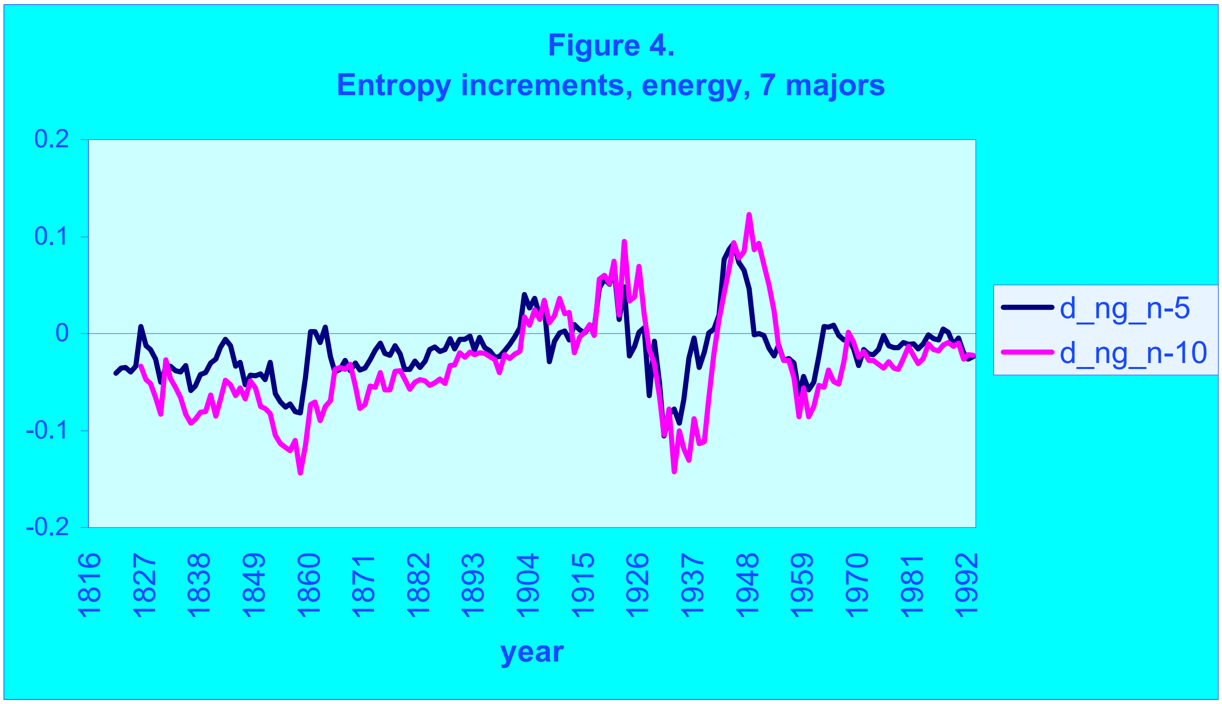


Figure 4.
Entropy increments, energy, 7 majors
0.2
0.1
0
-0.1
-0.2
1816
1827
1838
1849
1860
1871
1882
1893
1904
1915
1926
1937
1948
1959
1970
1981
1992
year
d_ng_n-5
d_ng_n-10

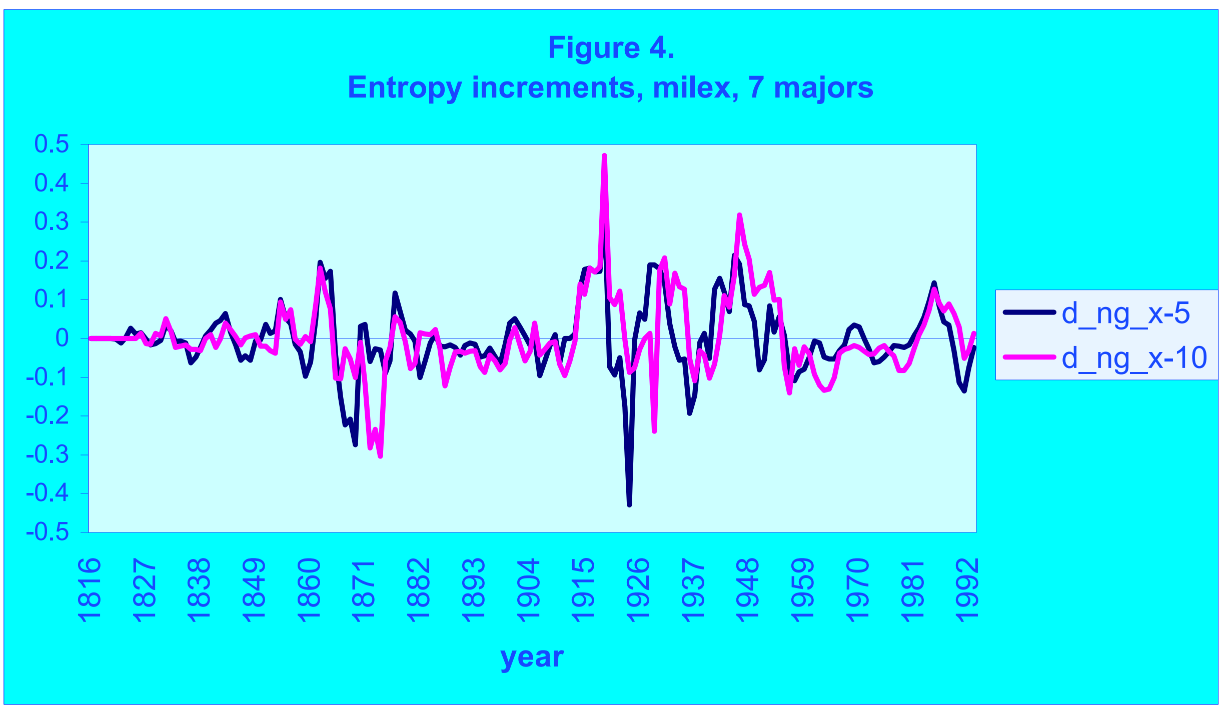


Figure 4.
Entropy increments, milex, 7 majors
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
1816
1827
1838
1849
1860
1871
1882
1893
1904
1915
1926
1937
1948
1959
1970
1981
1992
year
d_ng_x-5
d_ng_x-10

number	abbreviation	description
		1…5: change in log military personnel, indicated year from prior year.
1	d_l(m)t	referent year, t from year t-1
2	d_l(m)t-1	year t-1 from year t-2
3	d_l(m)t-2	year t-2 from year t-3
4	d_l(m)t-3	year t-3 from year t-4
5	d_l(m)t-4	year t-4 from year t-5
6	l(m)t-5	Log military personnel, year t-5.
		7…11: like 1…5, respectively, for log military expenditure.
7	d_l(x)t	referent year, t from year t-1
8	d_l(x)t-1	year t-1 from year t-2
9	d_l(x)t-2	year t-2 from year t-3
10	d_l(x)t-3	year t-3 from year t-4
11	d_l(x)t-4	year t-4 from year t-5
12	l(x)t-5	Log military expenditure, year t-5.
13	l(n)t	Log energy consumption, referent year t.
14	d_l(n)1-5	Change, log energy consumption, year t less year t-5.
15	l(p)t	Log total population, referent year t.
16	maj	= 1 if nation is COW project major power, referent year; else = 0.
17	sys	= 1 if nation is COW project system member, referent year t ; else = 0.
18	d_sys	[sys, year t] less [sys, year t-1]
		19…24: number of war starts, ref. nation
19	w-st-0	referent year t
20	w-st-1	year t-1
21	w-st-2	year t-2
22	w-st-3	year t-3
23	w-st-4	year t-4
24	w-st-5	year t-5
		25…30: like 19…24, respectively, for number of war terminations.
25	w-nd-0	referent year t
26	w-nd-1	year t-1
27	w-nd-2	year t-2
28	w-nd-3	year t-3
29	w-nd-4	year t-4
30	w-nd-5	year t-5
		31…36: like 19…24, respectively, for number of militarized interstate dispute (MID) starts, ref. nation.
31	cst-0	referent year t

32	cst-1	year t-1
33	cst-2	year t-2
34	cst-3	year t-3
35	cst-4	year t-4
36	cst-5	year t-5
		37…43: number of same-sided MID starts involving ref. nation with named nation in ref. year t ; with…
37	cst-sm_002	United States
38	cst-sm_200	United Kingdom
39	cst-sm_220	France
40	cst-sm_255	Prussia-Germany-West Germany
41	cst-sm_365	Russia
42	cst-sm_710	China
43	cst-sm_740	Japan
		44…50: number of opposite-sided MID starts involving ref. nation with named nation; with…
44	cst-op_002	United States
45	cst-op_200	United Kingdom
46	cst-op_220	France
47	cst-op_255	Prussia-Germany-West Germany
48	cst-op_365	Russia
49	cst-op_710	China
50	cst-op_740	Japan
		51…57: military personnel, named nation, prior to log transformation.
51	mlpr_002	United States
52	mlpr_200	United Kingdom
53	mlpr_220	France
54	mlpr_255	Prussia-Germany-West Germany
55	mlpr_365	Russia
56	mlpr_710	China
57	mlpr_740	Japan
		58…64: total number of same sided MID starts, named major with other 6 majors.
58	cts_002	United States
59	cts_200	United Kingdom
60	cts_220	France
61	cts_255	Prussia-Germany-West Germany
62	cts_365	Russia
63	cts_710	China
64	cts_740	Japan
		65…71: total number of opposite sided MID starts, named major with other 6 majors.
65	cto_002	United States
66	cto_200	United Kingdom
67	cto_220	France
68	cto_255	Prussia-Germany-West Germany
69	cto_365	Russia

70	cto_710	China
71	cto_740	Japan
		72, 73: change in negative entropy across 7 major powers, from year t-5 to referent year t, (Appendix 1) for named indicator…
72	d_ng_n-5	…for energy consumption
73	d_ng_x-5	…for military expenditure

Citations.

Correlates of War project. home page, http://www.umich.edu/~cowproj/

Global Vision, Inc. home page, http://www.globechange.org

Khinchin, A. I. (1957). *Mathematical Foundations of Information Theory*, trans. R. A. Silverman and M. D. Friedman. New York: Dover

Peace Science Society, International. home page, http://pss.la.psu.edu/

Richardson, Lewis F. (1960). *Arms and Insecurity*. Pittsburgh: The Boxwood Press and Chicago: Quadrangle Books.

Williamson, Paul. (1996). "Predicting Changes In USA Military Personnel Using Neural Network Modeling -- Two Approaches." Research Report 9601. Global Vision, Inc., P. O. Box 4394, Ann Arbor, MI 48106-4394.

Williamson, Paul and Bruce Bueno de Mesquita (2000). "Modeling the Number of United States Military Personnel Using Artificial Neural Networks." *Peace Economics, Peace Science and Public Policy*, 6, No. 4 (November 2000): 35-65, http://www.crp.cornell.edu/peps/Journal/Vol6-No4/Rsr-Williamson_deMesquita.pdf